INCORPORATING 21ST CENTURY COMPETENCIES IN MATHEMATICS TEACHING AND LEARNING AT THE JUNIOR SECONDARY LEVEL OF EDUCATION: A COLLABORATIVE ACTION RESEARCH STUDY

By
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A Development Oriented Research (DOR) study supported by the Accelerating Higher Education Expansion and Development (AHEAD) operation of the Ministry of Higher Education, Sri Lanka, and the World Bank.

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List of Abbreviations

21CC	-Twenty first Century Competencies
AR	-Action Research
ATC21S	-Assessment and Teaching of 21st Century Skills
CAR	-Collaborative Action Research
CBAR	-Classroom Based Action Research
CBC	-Competency Based Curriculum
COVID-19	-Corona Virus Disease 2019
CPA	-Concrete-Pictorial-Abstract
CPS	-Collaborative Problem Solving
DeSeCo	-Definition and Selection of Competencies
GCE(O/L)	-General Certificate of Education Ordinary Level
HCF	-Higest Common Factor
IC	-Instructional Core
ICT	-Information Communication Technology
ISA	-In-Service Advisor
ISAs	-In-Service Advisors
ISTE	-International Society for Technology in Education
JS	-Junior Secondary
JSC	-Junior Secondary Curricula
LCM	-Least Common Multiple
MoE	-Ministry of Education
NCREL	-North Central Regional Educational Laboratory
NCTM	-National council for teachers of Mathematics
NEC	-National Education Commission
NIE	-National Institute of Education
NRC	-National Research Council
OECD	-Organization for Economic Co-operation and Development
P21	-Partnership for 21st Century Skills
PDE	-Provincial Department of Education
PDE	-Provincial Department of Education
PGIHS	-Postgraduate Institute of Humanities and Social Sciences

PISA	-Programme for International Students Assessment
SBTPD	-School Based Teacher Professional Development
TIM	-Teacher Instructional Manual
TSLN	-Thinking School Learning Nation
UNESCO	-United Nations Educational, Scientific and Cultural Organization
Wi-Fi	-Wireless Fidelity

Abstract

The current Collaborative Action Research (CAR) study is the second phase of the three-year study, titled 'Mathematics Education for the 21st Century: A study of improving teaching and learning in mathematics at the junior secondary level in Sri Lanka'. The diagnostic study implemented in the phase 1 revealed many issues affecting student learning in mathematics classrooms at the junior secondary level in the Central province. These issues must be addressed at multiple levels of the education system. Since we are from a university department of education , which is responsible for teacher education and professional development , we were motivated to address the issues of student learning and teachers' teaching at the classroom level. The diagnostic study revealed that about 73% of students scored below 40 marks at the first term test in 2019, which was conducted by the provincial department of education. Around forty percent of students reported mixed feelings towards mathematics and classroom observations revealed that all five standards used to assess teachers' classroom practices need improvements. We decided to address these issues and improve student learning and 21st Century Competencies (21CC) using a CAR approach that focused on a sample of classrooms and teachers. Four university researchers collaborated with six teachers, one In-Service Advisor (ISA) and an education officer in the CAR process. The CAR implemented in three cycles, which addressed the following three key inquiry questions. 1. How can we introduce 21CC into mathematics teaching and learning in the Junior secondary level classrooms through CBAR by teachers? 2. How can we facilitate teacher professional learning through CBAR? 3. How effective is the CAR and the CBAR implemented by teachers/ISAs/and officers and how can we share our understandings with important others? We have started the process by conducting a workshop for resource persons by Prof B. Kaur (NIE, Singapore). Then we recruited collaborating teacher and officer researchers after a dissemination seminar held to share the findings of Phase 1 study with the provincial and zonal officers, principals, Mathematics teachers and ISAs. Subsequently, four initial workshops held in person on incorporating 21st century thinking and learning skills into mathematics teaching and learning process and using Classroom Based Action Research (CBAR) for improving student learning and teacher learning. At the end of first cycle the teacher and officer collaborators presented their CBAR proposals and received feedback from the university collaborators. Second cycle focused on implementing CBAR, progress review and providing guidance and feedback. Based on the reflections of the university team, further six workshops had been implemented online to provide necessary theoretical and conceptual inputs to the collaborators during the second cycle. Third cycle focused on the evaluation and reflecting on the CBAR and CAR, writing CBAR reports by collaborating teachers and officers, dissemination of findings of CBAR and CAR through participation in international and national research conferences, writing journal articles/ conference papers and publication of 2 digital books. Evaluations and reflections on the CBAR and CAR helped us to develop a socially situated model of CAR for incorporating 21CC in mathematics classrooms and improving student learning while facilitating teacher professional learning. Implications of our findings and the model for incorporating 21CC in mathematics classrooms for policy, practice and research are discussed and conclusions are presented at the end of final chapter.

Key words: Collaborative Action Research, Classroom Based Action research, 21st Century Competencies, Mathematics, Teaching and learning.

Chapter 1: The context and the rationale of the study

1.0 Introduction

The current Collaborative Action Research (CAR) study is the second phase of the three-year study, titled 'Mathematics Education for the 21st Century: A study of improving teaching and learning in mathematics at the junior secondary level in Sri Lanka'. In phase 1, a diagnostic survey research study conducted in 50 schools in the Central province revealed that many students achieved poorly in mathematics at the term test conducted by the Provincial Department of Education (PDE). The reasons for the poor achievements of the students are related to many factors of schools, students, teachers, curriculum, teaching, and assessments. The situation demands many interventions at the classroom, school, zonal and provincial education, and the National levels (MoE and NIE). However, as academics from a Department of Education responsible for teacher education and professional development, we were interested in intervening at the classroom level through mathematics teachers, officers, and ISAs by improving teacher professional learning and students' outcomes. Therefore, in Phase 2 of the study, we have collaborated with 17 mathematics teachers, officers and ISAs who were willing to collaborate with us to explore possibilities for improving student learning and achievements through incorporating 21st Century competencies among the learners. The interventions had been planned to be implemented over a 9–12-month period during 2020 and 2021. However, due to the COVID-19 pandemic, we had to implement it over a two year period, spanning from July 2020 to July 2022 with many interruptions resulting from periodic lockdowns, school closures, trade union action of the teachers during 2021 and economic and political crisis in 2022. The majority of initially planned 10 workshops and progress review meetings had to be conducted using the virtual mode of interactions. Due to connectivity issues and other difficulties faced by the participants, the total number of participants who continued to complete the project dropped to 8 during this period. However, despite these constraints we managed to achieve our purpose of developing a group of committed teachers and officers who are perseverant and competent in conducting classroom-based action research to improve students learning through instilling 21st century skills in their mathematics classrooms. In this book our purpose is to discuss the context and the process of CAR that we adopted, and the model of CAR evolved in the process. To achieve this purpose, we wrote five chapters. In Chapter 1 we present the context of the study, where we describe the rationale of the current CAR study and the importance of incorporating 21st Century Competencies(21CC) in the total curriculum at school level and in the mathematics curricula, using current literature available on 21CC. Chapter 2 is devoted to explaining the methodology adopted in this study. In Chapter 3, we describe the details of the workshop conducted for the resource persons by Prof. Berinderjeet Kaur of National University of Singapore on incorporating 21CC in mathematics classrooms. Chapter 4 describes in detail the implementation of the CAR and finally, in Chapter 5 we present our evaluations and reflections and the model of CAR evolved and its' implications for policy practice and research. We believe the experiences that we gained in the process and the insights that we report in the CAR would be useful to the readers of this document to design and implement CAR in future to improve student learning and achievements particularly in mathematics and in other subjects.

In the current chapter we discuss the context of the study and examine the need to incorporate 21CC in the mathematics curricula and the teaching learning and assessment practices at the JS level in Sri Lanka.

1.1 The context of the CAR study

Education should endeavour to prepare children to adapt to the rapidly changing world of the future and to empowering them to actively engage in making it better. However, growing evidence from research, and widespread public opinion indicate that education systems are far from achieving this purpose. Students are often not adequately prepared to succeed in today's world, let alone the world of future (Fadel et al, 2015). Therefore, education needs fundamental reforms from top to bottom to prepare students for the 21st century requirements.

Our current circumstances require new models of education which are developed using participatory approaches and research-based evidence to adapt to the demands of the 21st century. For this purpose, we have designed a CAR study, which focuses on the need for reforms at classroom level in teaching and learning Mathematics to instil 21st century competencies among students at the Junior Secondary (JS) level of education in Sri Lanka to improve students' achievements in mathematics. The study specifically attempts to provide useful evidence on teaching and learning of Mathematics at classroom level to inform education reforms and policy at different levels of the education system. The study has been conducted in two phases. Phase 1 was a survey research study designed to diagnose the existing situation of students' mathematics achievements and the factors associated with students' achievements. Phase 2 was a CAR conducted in a smaller sample of schools involving mathematics teachers, In-Service Advisors (ISAs) and officers responsible for Mathematics education in the Central Province.

The study implemented in the Central province of Sri Lanka during the period from January 2019 to July 2022. The CAR study is based on the key findings of phase 1 study.

First and foremost, of our findings in Phase 1 are the poor student achievements in mathematics. Data collected from 50 schools in the central province revealed that students' achievements are far below the expected level. Figure 1 presents the cumulative frequency percentages of the mathematics scores of the students in the first term test conducted by the Provincial Department of Education (PDE) of the Central province in 2019.

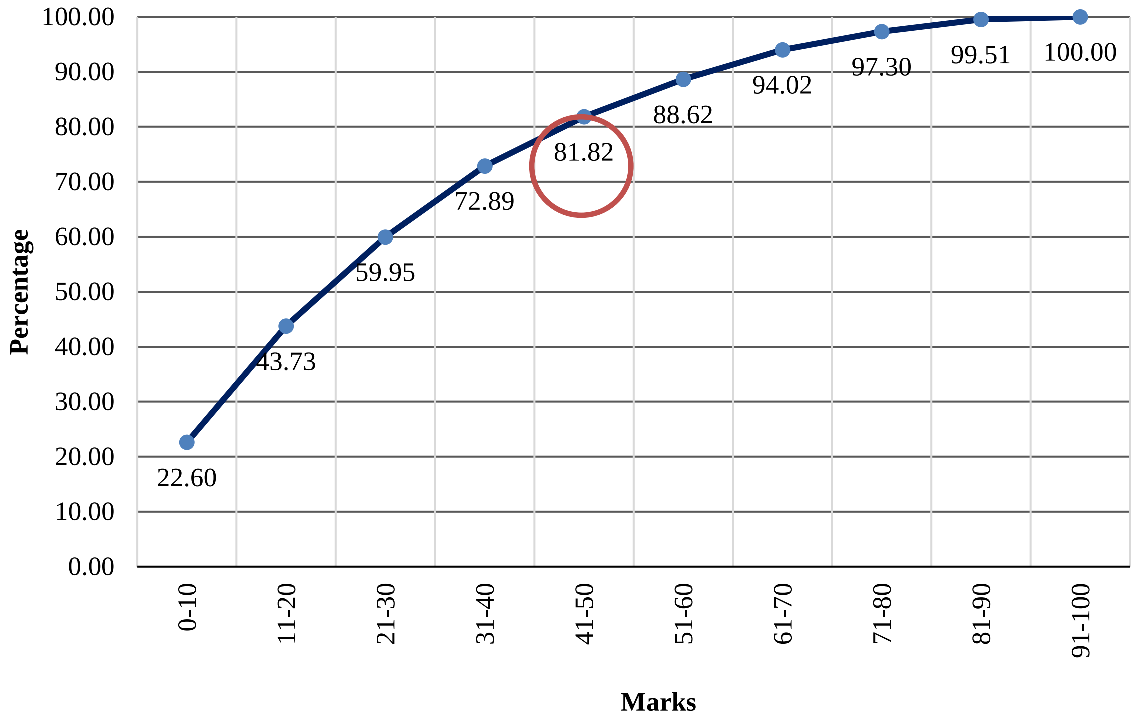


Figure 1: Cumulative frequency percentages of Mathematics scores at Grade 7

According to Figure 1, about 73% of students scored below 40 marks and about 23% scored below 10 marks.

The findings of Phase 1 study revealed that insufficient physical and human resources (especially the professionally qualified mathematics teachers), and issues related to teacher education and professional development, teachers' classroom practices, students' negative attitudes towards mathematics, students' absenteeism, the lack of use of stimulating learning materials, and teacher beliefs affect this situation.

The issues related to insufficient physical and human resources, and teacher education and professional development needs to be addressed at the national, provincial, and school levels. However, we believe that the issues related to teachers' classroom practices, students' negative attitudes towards mathematics, students' absenteeism, the lack of use of stimulating learning materials, and teacher beliefs can be addressed more effectively at the school and classroom levels through other measures such as Classroom Based Action Research (CBAR). Therefore, in phase 2 of the study, we focused on the following key aspects that need to be addressed to improve students' learning:

1.Teachers' classroom practices

Figure 2 indicates the percentages of teachers' adherence to quality standards measured using a standardised observation schedule. According to the data, classroom practices related to all six standards measured in the study need improvement while the use of adaptive teaching, teaching learning strategies that promote transfer of learning and mastery, as well as creating safe and stimulating learning environments in classrooms need specific attention.

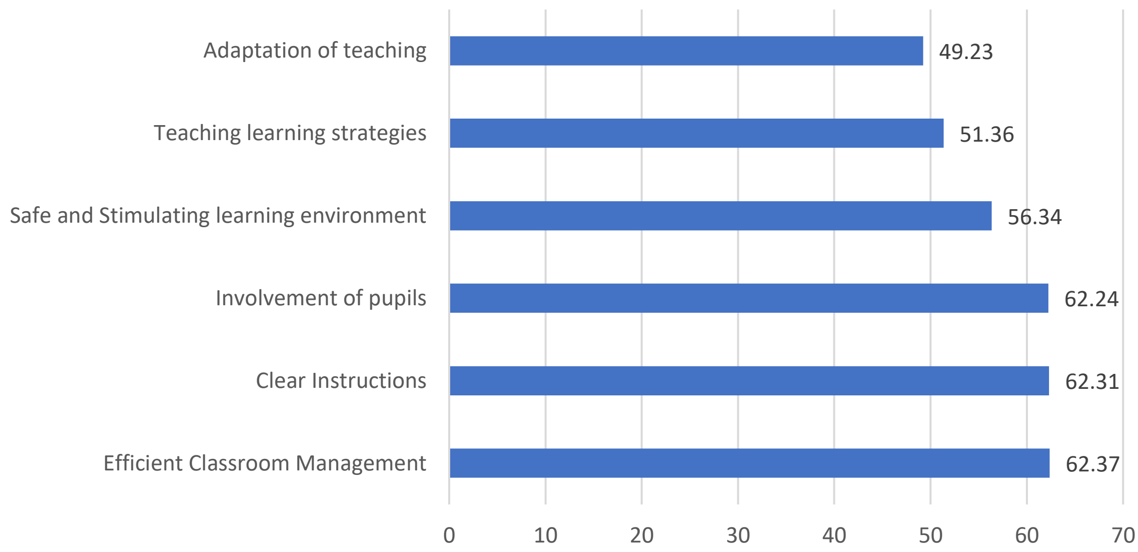


Figure 2: Percentages of quality standards adhered to by teachers

Further analysis of the best practices used by the teachers and qualitative observations revealed that the use of effective classroom management practices, which provide opportunities for students to interact with peers and collaborative learning and scaffolding also need improvement.

2. Teacher Beliefs

Four main themes of teachers' beliefs of students' low achievement emerged in the analysis of data collected through interviews with teachers. They are students' related factors; home environment related factors; school related factors and curriculum related factors. Teachers attributed students' insufficient prior knowledge, poor attitudes and motivations, insufficient support for learning mathematics at home, lack of mathematics labs and other resources at schools, and weaknesses in the curriculum to the students' low achievements. Teachers focused on insufficient human and physical resources and issues such as students' lack of adequate prior knowledge, motivation, and support from home environment in their views on current strategies, and suggested strategies to overcome poor achievements. We felt that the issues on human and physical resources must be addressed at the other levels of education administration while students related issues must be addressed at the school and classroom levels.

The above findings clearly indicate that students' poor achievements in mathematics must be addressed at classroom level by paying attention to changing teacher beliefs and practices. We believe that, if the teachers are encouraged to take action to improve the situation and supported adequately by the peers and other stakeholders, they would be able to design and implement more context-based solutions for improving student outcomes. The reflective actions implemented by teachers in classroom-based action research will shape teacher beliefs (Menfra, 2019) and we argue that such actions can be used to develop favourable attitudes towards learning mathematics among students and to improve their motivations, values and achievements. Moreover, we were interested in incorporating 21st Century Competencies (21CC) into the mathematics teaching and learning, since the students need to master these competencies to become successful and engaged citizens in a rapidly changing technological, cultural, social, and economic environment of the 21st Century. Based on these two premises we have designed and implemented this CAR study. Before describing our methodology in Chapter 2, we review literature on the 21CC and the 21CC frameworks in the next sections of the current chapter to underpin the need to incorporate 21CC into the mathematics curricula, and teaching, learning and assessment practices in Sri Lanka.

1.2 Twenty first century competency frameworks

The 21st century is quite different from the 20th century in relation to the social, economic, and technological developments and the capabilities people need for work, citizenship, and to succeed in life. The emergence of digital revolution characterized by personal, mobile, and networked technologies has replaced manual and routine mental labour with ideas, innovation and personalized services that drive economic growth and social mobility (Tan et al, 2017; Dede, 2006). The new situation demands the education systems to adapt and respond to the evolving human capital requirements of industry and workplaces, and to the learning needs of the students.

According to the Ministry of Education (MoE, 2016), the call for education systems to focus on such reforms is linked to the following 3 developments:

1. Changes in the work force from an industrial model of production to a rapidly transforming, technology-driven, and interconnected globalized knowledge economy. Such an economy requires competencies suited to dynamic and unpredictable models of economic and social development.
2. Emerging evidence on how to optimize learning, including the use of technological innovations to deepen and transform learning; and
3. Changing expectations on the part of learners, who are demanding an education system that is more connected and relevant to their everyday lives (MoE, 2016, p.7).

The demands arising from these developments led the national governments and international education communities to make a concerted effort on two educational endeavours. One, to identify 21st Century Competencies (21CC) that young people require to become active designers of, and productive contributors to local and global futures of political, social, cultural, and economic development. Two, to identify effective teaching, learning and assessment strategies that can be used in formal and informal educational contexts to develop such competencies among young people (Tan et al, 2017). Review of literature indicates that although there is no agreement about specific school curricula to be adopted to achieve the relevant outcomes by the students, there is some agreement among the practitioners, researchers, and policymakers about the constituents of 21CC and the teaching, learning and assessment strategies that are likely to guide the learners to achieve them (Tan et al, 2017).

The MoE (2016) surmises that international organisations and groups like the OECD, the European Commission, the Partnership for 21st Century Skills (P21), and the U.S. National Research Council have been instrumental in bringing about rigor to the research and intellectual debate on 21st century competencies. It further observes that similar conceptual understandings of the competencies are reflected in the frameworks developed by different countries such as Australia, England, Finland, Japan, Northern Ireland, Scotland, and Singapore, and individual researchers such as, Fullan, Jenson and Dede, as well as the International/National organisations, listed below:

Assessment and Teaching of 21st Century Skills (AT21CS)
Partnership for 21st Century Skills (P21)
Canadians for 21st Century Learning (C21 Canada)
Association of American Colleges and Universities
European Commission
International Society for Technology in Education (ISTE)
Canadian provinces of Alberta, British Columbia, and Quebec
National Academy of Sciences (National Research Council)
Organisation for Economic Co-operation and Development (OECD)
North Central Regional Educational Laboratory (NCREL)
U.S. Department of Labour (MOE, 2016, p.9)

Tan et al (2017) compare five frameworks of 21CC developed by National Academy of Sciences, Partneship for 21st Century skills (P21), Assessment and teaching of 21st century skills (ATC21S), OECD and European Union(EU). Closer look at Table 1 gives us an idea about the commonalities and variations in the ways of categorising different competencies. Although these different frameworks use different categories and terminologies, they all emphasise certain set of cognitive, personal (Intrapersonal) and social (Interpersonal) skills or competencies. The comparisons of different 1st century skills frameworks indicate that the competencies are generally consistent across the frameworks.

Table 1: An overview of international 21CC education frameworks

National Academy of Sciences' Education for Life and Work: Developing Transferable 21st Century Knowledge and Skills	Partnership for 21st Century Skills (P21)	Assessment and Teaching of 21st Century Skills (ATC21S)	OECD Definition and Selection of Competencies (DeSeCo)	EU Key Competences for Lifelong Learning
Cognitive competencies	**Learning and innovation skills**	**Ways of thinking**	**Using tools interactively**	**Learning to learn**
Cognitive processes and strategies. Knowledge. Creativity.	Creativity and innovation. Critical thinking and problem-solving.	Creativity and innovation. Critical thinking, problem-solving, decision-making. Learning to learn, meta-cognition.	Use language, symbols, and texts interactively. Use knowledge and information interactively. Use technology interactively.	Sense of initiative, entrepreneurship.
	Information, Media, and technology skills	**Tools for Working**		**Mathematical competence and basic competence in science and technology**
	Information literacy. Media literacy. ICT literacy.	Information literacy. ICT literacy.		Digital competence.
Interpersonal competencies	**Learning and innovation skills**	**Ways of Working**	**Interacting in Heterogeneous Groups**	**Communication in mother tongue**
Teamwork. Leadership.	Communication. Collaboration.	Communication. Collaboration, teamwork.	Relate well with others. Co-operate, work in teams. Manage and resolve conflicts.	
Intrapersonal competencies	**Life and career skills**	**Living in the World**	**Acting Autonomously**	**Social and civic competencies**
Intellectual openness. Work ethic, conscientiousness. Positive core self-evaluation.	Flexibility, adaptability. Initiative, self-direction. Social, cross-cultural skills. Productivity, accountability. Leadership, responsibility.	Citizenship (local and global). Life and career skills. Personal and social responsibility. Personal and social responsibility (Including cultural awareness and competence).	Act within big picture. Form and conduct life plans and personal projects. Defend and assert rights, interests, limits and needs.	Cultural awareness and expression.

Source: Tan et al (2017)

1.2.1 Twenty first Century skills and Competencies

The competencies that the 21st century learners need to possess include an array of skills, values, and practices – such as critical thinking, creativity, communication, and respect for diversity, adaptability, entrepreneurship, and innovation. Nomenclature of these competencies varies and includes terms such as 'twenty-first century skills', 'non-cognitive skills' and 'non-academic skills. Although there is no consensus on the terminology, the importance of these competencies is widely accepted (UNESCO, 2016).

The frameworks compared in the above seem to use the terms skills and competencies interchangeably. However, the term competency is broader and more inclusive concept than the term skills (MoE, 2016). In the following quotation (OECD, 2003) posits that competency is more than just knowledge or skills and distinguishes the differences between skills and competencies.

"A competency is more than just knowledge or skills. It involves the ability to meet complex demands, by drawing on and mobilising psychosocial resources (including skills and attitudes) in a particular context. For example, the ability to communicate effectively is a competence that may draw on an individual's knowledge of language, practical IT skills and attitudes towards those with whom he or she is communicating." (OECD, 2003, p. 4)

Cedefop glossary of the European commission defines "skill" as *the ability to perform tasks and solve problems*, and "a competency" as *the ability to apply learning outcomes adequately in a defined context* (for example, education, work, personal or professional development) (Cedefop, 2014).

According to the above definition, competency is not limited to cognitive abilities (involving the use of theory, concepts, or tacit knowledge) but includes both practical abilities (involving technical skills) and interpersonal skills (e.g., social, or organizational skills) as well as ethical values. A competency is therefore a broader concept that may comprise knowledge, skills, ethical values as well as attitudes (MoE, 2016).

Since the different frameworks use the terms skills and competencies interchangeably, in this introductory chapter we use both terms of skills and competencies interchangeably in describing the 21st century competencies, however, in the rest of the chapters we use the term competencies consistently in its precise meaning highlighted in the above.

The cognitive competencies of critical thinking, creativity, communication, and problem solving have been regarded as key factors for success in life and career for a long time.

However, changing economic, technological, and social contexts in the 21st century, demands new interpretations of these competencies and innovative pedagogical approaches to teach and assess them in formal and informal settings (Dede, 2010). Moreover, rapid changes occurring in these contexts make the interpersonal and intrapersonal competencies much more important than they were in the past. Employers are increasingly valuing "soft" skills such as teamwork and leadership skills. Furthermore, research evidence suggests that young people's social skills affect their job prospects in adulthood (Tan et al,2017).

The above review indicates that many countries around the world have been already focusing on 21CC, because of the demands arising from the rapidly changing, technology-driven, and interconnected globalised knowledge economy. Sri Lanka, also need to focus on developing young generations capable of facing the challenges of the 21st century, achieving personal success and becoming productive and engaged citizens to make our country a better place. The education system of the country must play a key role in this endeavour by developing a suitable curriculum framework, student outcomes and pedagogical strategies.

1.2.2 The need to incorporate 21CC in mathematics teaching, learning and assessment practices in Sri Lanka

The purpose of this section is to discuss the importance of incorporating 21st CC in the teaching learning process mathematics at the junior secondary level. Here, we analyse, how the 21st century skills are currently represented and implemented in the junior secondary mathematics curriculum in Sri Lanka and the importance of incorporating such skills into the mathematics curriculum, teaching learning and assessment practices.

During the first two decades of the 21st century, Sri Lanka has implemented two sets of policy reforms in Education in 1997 and 2006. Eight National goals for education and five competencies have been introduced in 1997 reforms. In 2006 the number of competencies has been increased to seven and subsequently, a Competency Based Curriculum (CBC) was gradually introduced during the period from 2007 to 2011. The seven National competencies advocated in the CBC included.

1. Competencies in Communication (based on four subsets: Literacy, Numeracy, Graphics and IT proficiency}.

2. Competencies relating to Personality Development
    - Generic skills such as creativity, divergent thinking, initiative, decision making, problem solving, critical and analytical thinking, teamwork, inter-personal relations, discovering and exploring.
    - Values such as integrity, tolerance, and respect for human dignity.
    - Emotional intelligence.

3. Competencies relating to the Environment: These competencies relate to the environment: social, biological, and physical.

4. Competencies relating to Preparation for the World of Work: Employment related skills to maximize their potential and to enhance their capacity
    - to contribute to economic development,
    - to discover their vocational interests and aptitudes,
    - to choose a job that suits their abilities, and
    - to engage in a rewarding and sustainable livelihood.

5. Competencies relating to Religion and Ethics: Assimilating and internalizing values, so that individuals may function in a manner consistent with the ethical, moral and religious modes of conduct in everyday living, selecting that which is most appropriate.

6. Competencies in Play and the Use of Leisure

7. Competencies relating to "learning to learn"
   Empowering individuals to learn independently and to be sensitive and successful in responding to and managing change through a transformative process, in a rapidly changing, complex and interdependent world. (NEC, 2006).

These competencies seem to match with most of the 21CC defined by other countries and in international literature. One important missing element is metacognition which is an essential component of effective learning and problem solving.

In the teacher instructional manuals, the rationale for identifying, or the significance of the above competencies in the 21st century classroom learning and the specific pedagogical and assessment approaches that should be adopted to infuse these competencies among the learners are not adequately elaborated. For instance, a published research study that examined whether the CBC has achieved its objectives reveal that it has not fulfilled its objectives as a competency-based curriculum (Egodawatta, 2014). The researcher concludes that 'competency-based teaching and learning approaches were superficially introduced in mathematics education in Sri Lanka and the curriculum documents did not properly reflect their intended objectives'. McCaul (2007) also made a similar conclusion when he studied the mathematics curriculum from Grade 6-11 in Sri Lanka and states that 'the syllabus does not include learning outcomes for the process standards of Communication, Relationships, Reasoning and Problem Solving. Intended learning outcomes in these standards can be interpreted through an analysis of the activities in the TIM but they are not set out in the descriptive syllabus as a way of emphasizing their importance in the teaching of mathematics and evaluating the student learning'(McCaul, 2007, p. 46).

This situation led us to peruse the internationally published literature to study how the other countries identified key competencies to be achieved in the 21st Century and what pedagogical approaches that they advocate to facilitate the students to acquire such competencies. A good example could be found in a Ministry of Education (MoE), Ontario, Canada publication on *Towards Defining 21st Century Competencies for Ontario*. Further exploration of literature reveals that almost all available international frameworks (e.g. Singapore, Partnership for 21st Century skills (P21), Assessment and Teaching of 21st Century Skills (ATC21S) and OECD-Definition and Selection of Competencies (DeSeCo)) have been developed using extensive reviews of research literature on competencies and appropriate pedagogical approaches as well as wider discussions held among different stakeholders. Another important observation one can make is the effort made by the designers to explicate the definitions and the rationale behind each competency included in the framework as well as the elaboration of teaching learning and assessment strategies that needs to be used to assist the students to develop these competencies (See for example, P21- *21st Century Skills Map* available at https://files.eric.ed.gov/fulltext/ED543032.pdf).

A study conducted by UNESCO, in Asia Pacific countries (including: Australia, China , India, Japan, Malaysia, Mongolia, Republic of Korea, Thailand and Viet Nam) reveal that all these countries have incorporated 'Transversal competencies' into the school curricula. Transversal competencies are also called 21CC. In these countries, five competencies that include (i) critical and innovative thinking; (ii) inter-personal skills; (iii) intra-personal skills;

(iv) global citizenship; (v) media and information literacy skills are incorporated into the school curricula through the following three different ways:

1. Specific subject: the competencies are included as a well-defined entity within the formal curriculum, for example, a subject with specific goals and syllabus for formal teaching.

2. Cross subject: Transversal competencies are introduced across 'vertical subjects' (i.e., traditional school subjects) or they infiltrate and/or underpin them.

3. Extracurricular activities: Transversal competencies are made part of school life and are embedded purposefully into all types of non-classroom activities. UNESCO (2016, p.18)

The study also reveals three main challenges faced by the teachers in fully integrating transversal competencies:

1. **Definitional challenges**, arising from a lack of, or a vague definition of, transversal competencies in policy documents.
2. **Operational challenges**, such as a lack of adequate evaluation systems for transversal competencies.
3. **Systemic challenges**, including inconsistency between transversal competencies in the curriculum and the contents of the existing high-stake examinations, especially for university entrance examinations (UNESCO, 2016, p.8).

These findings, although from an international study, are useful in designing and implementing a new curricular framework incorporating 21CC in Sri Lanka since the issues highlighted in them are comparable to those in the Sri Lankan context. Based on these observations we can conclude that Sri Lanka also need to further develop and elaborate its own curricular framework of 21CC together with appropriate teaching, learning and assessment approaches that need to be adopted in delivering the curricula as discussed in the remaining section of this chapter.

The current junior secondary mathematics curriculum in Sri Lanka is based on an adapted version of the content standards and process standards specified by the National Council for Teachers of Mathematics (NCTM) which are widely used internationally in designing mathematics curricula (McCaul, 2009). NCTM (2000) defines ‘standards’ as the mathematical content and processes that students ‘should know’ and ‘be able to do or use’ as they progress from pre-kindergarten to Grade 12 mathematics. There are five **content standards** and five **process standards**. The five **content standards** include:

- Number and operations
- Measurement
- Geometry
- Algebra
- Data analysis and probability

The process standards include:

- Communication
- Representation
- Connections
- Reasoning and proof
- Problem solving

According to the Grade 7 Mathematics Teacher Instructional Manual (TIM) the aim of learning mathematics at the junior secondary level is to further develop the mathematical concepts, creativity, and sense of appreciation in students entering the junior secondary stage, so that their mathematical thinking, understanding, and abilities are formally enhanced. To achieve this aim five objectives have been set. The objectives seem corresponding to the process standards specified in the ‘Principles and standards for teaching mathematics’ published by the NCTM in the year 2000.

Table 2 sets out the process standards specified by the NCTM (2000) and the objectives of learning mathematics at the junior secondary level (NIE, 2016).

Table 2: Process Standards (NCTM) and the objectives of learning mathematics

Process standards (NCTM, 2000)s	**Corresponding Objectives of learning Mathematics (Gr 7-TIM, 2015)**
1. **Representations.** Mathematical ideas can be represented in a variety of ways: pictures, concrete materials, tables, graphs, number and letter symbols, spreadsheet displays, and so on. The ways in which mathematical ideas are represented is fundamental to how people understand and use those ideas. Many of the representations we now take for granted are the result of a process of cultural refinement that took place over many years. When students gain access to mathematical representations and the ideas they express and when they can create representations to capture mathematical concepts or relationships, they acquire a set of tools that significantly expand their capacity to model and interpret physical, social, and mathematical phenomena.	1. **Knowledge and skills** The development of computational skills through the provision of mathematical concepts and principles, as well as knowledge of mathematical operations, and the development of the basic skills of solving mathematical problems with greater understanding.
2. Communication. Mathematical communication is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language.	2. **Communication** The development of correct communication skills by enhancing the competencies of the proper use of oral, written, pictorial, graphical, concrete, and algebraic methods.
3. **Connections.** Mathematics is not a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study. When students connect mathematical ideas, their understanding is deeper and more lasting, and they come to view mathematics as a	3. **Relationships** The development of connections between important mathematical ideas and concepts, and the use of these in the study and improvement of

coherent whole. They see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. Through instruction that emphasizes the interrelatedness of mathematical ideas, students learn not only mathematics but also about the utility of mathematics.	other subjects. The use of mathematics as a discipline that is relevant to lead an uncomplicated and satisfying life.
4. **Reasoning and Proof**. Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena. People who reason and think analytically tend to note patterns, structure, or regularities in both real-world and mathematical situations. They ask if those patterns are accidental or if they occur for a reason. They make and investigate mathematical conjectures. They develop and evaluate mathematical arguments and proofs, which are formal ways of expressing particular kinds of reasoning and justification. By exploring phenomena, justifying results, and using mathematical conjectures in all content areas and—with different expectations of sophistication—at all grade levels, students should see and expect that mathematics makes sense.	**4**. **Reasoning** The enhancement of the skills of inductive and deductive reasoning to develop and evaluate mathematical conjectures and conversations.
5. Problem Solving. Solving problems is not only a goal of learning mathematics but also a major means of doing so. It is an integral part of mathematics, not an isolated piece of the mathematics program. Students require frequent opportunities to formulate, grapple with, and solve complex problems that involve a significant amount of effort. They are to be encouraged to reflect on their thinking during the problem-solving process so that they can apply and adapt the strategies they develop to other problems and in other contexts. By solving mathematical problems, students acquire ways of thinking, habits of	**5. Problem solving** The development of the ability to use mathematical knowledge and techniques to formulate and solve problems, both familiar and unfamiliar and which are not limited to arithmetic or the symbolical or behavioral, which arise in day today life.

persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom. (Source: NCTM(2000))	

One of the weaknesses that we see in the Teacher Instructional Manuals (TIM) is that they do not provide a clear explanation to teachers about the rationale behind the objectives (which are based on process standards) and how those objectives are related to the prescribed contents, competencies, learning outcomes and tasks in the TIM.

A careful comparison of the process standards (NCTM, 2000) and the objectives of learning mathematics at the junior secondary level (NIE, 2016) shows commonalities as well as some differences between them. There are some differences in the terminology used in the two sets of Standards and objectives in Table 2. For example, in the objectives the process of mathematical representation is not defined separately in the Junior Secondary Curricula (JSC). However, the process of representation is implicit in both objectives 1 and 2. The process of communication is also defined slightly differently in the objective 2 of JSC compared to the NCTM process standard 2. The NCTM standard 2 defines the process of communication while emphasizing the communication of mathematical ideas by students in both oral and written forms during the classroom learning process. Process standards (NCTM, 2000) also emphasizes the thinking, reasoning, representation, and communication processes and define them more elaborately than the objectives of the JSC.

In a study conducted by the NIE, Sri Lanka, McCaul (2007) states that the objectives of junior secondary mathematics curriculum and standards of mathematics (namely, Knowledge and skills, Communication, relationships, reasoning, and problem solving) are directly aligned with international trends. International curricula place emphasis on learning the methods and tools of mathematics, relating mathematics to other subject areas, and developing skills to solve everyday problems using mathematics.

McCaul (2007) compares the process standards elaborated in Grades 6 and 10 curricula in Sri Lanka with NCTM content and process standards and make the following important observations.

1. The curricula provide a clear accounting of the content standards to be taught along with learning outcomes that focus on basic mathematics skills.

2. The syllabus does not include learning outcomes for the process standards of Communication, Relationships, Reasoning and Problem Solving. Intended learning outcomes in these standards can be interpreted through an analysis of the activities in the TIM but they are not set out in the descriptive syllabus as a way of emphasizing their importance in the teaching of mathematics and evaluating the student learning. (McCaul, 2007, p. 46)

At the end of his detailed analysis of the Grade 6 and 10 mathematics curricula McCaul (2007) provides seven recommendations for improving the descriptive syllabi and TIMs. These recommendations highlight the need to integrate the process standards of communication, relationships, reasoning, and problem solving into the current organization structure of the curriculum which is based on the content standards of numbers, measurement, geometry, algebra, and statistics, sets and probability. McCaul's recommendations highlight the importance of specification of relevant learning outcomes that are aligned with content and process standards, appropriate mathematical tasks and assessment of students learning to improve mathematics curricula at JS level.

According to MoE (2016) the most prominent 21st century competencies found in international frameworks that have been shown to offer measurable benefits in multiple areas of life are associated with critical thinking and problem solving, communication, collaboration, and creativity and innovation. Learning and Innovation skills presented in P21 framework name these competencies as 4Cs. (See Figure 3)

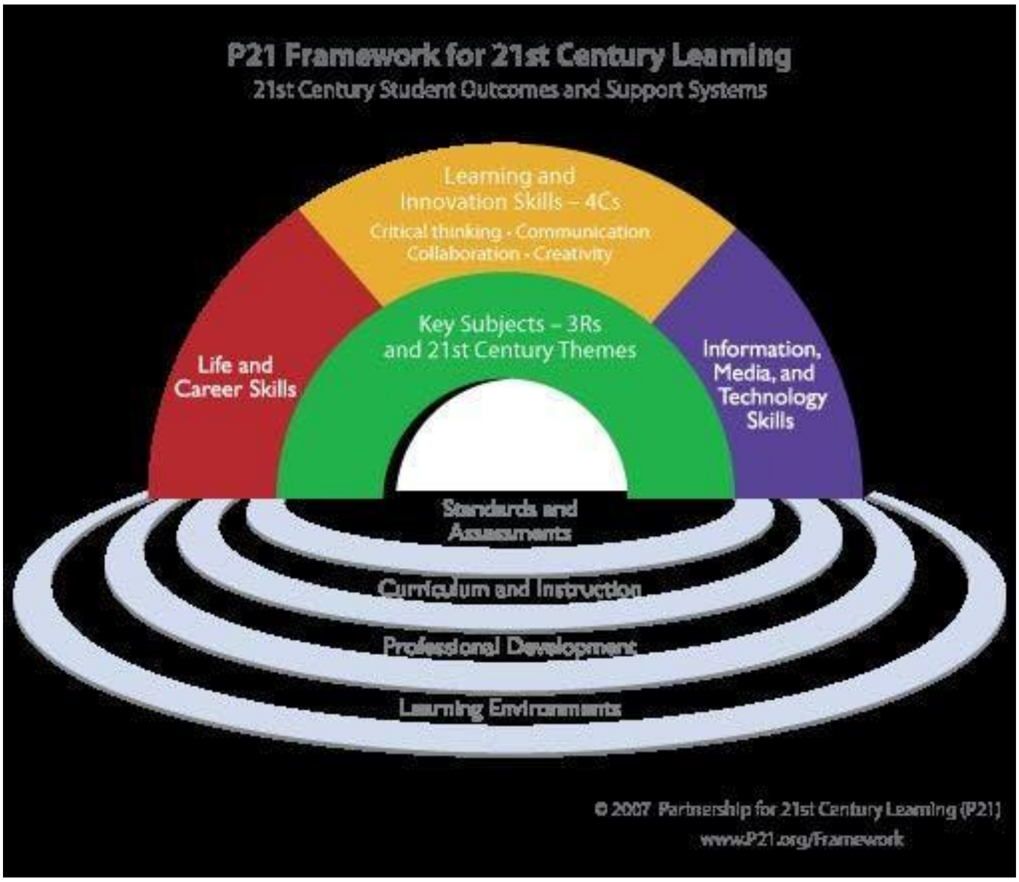


Figure 3: P21 framework for 21st Century Learning-21st Century students' outcomes and support Systems

Source: Partnership for 21st Century Learning(P21) www.p21..org/Framework

Comparison of these skills and NCTM process standards indicate that the latter include those skills in different standards and in the objectives of learning mathematics. For example, although the term creativity is not directly used in any of the standards, however, problem solving, representation, relationships and communication involve creativity. Moreover, ability to see connections between different elements, ideas and objects enhances one's creativity. Similarly critical thinking is necessary in observing and using connections, reasoning and proof, problem solving, communication and representation. Therefore 21st century learning, and innovation skills can be integrated into the JSC through a deliberate attempt of defining relevant outcomes, teaching, and learning strategies and assessments and empowering teachers to incorporate those skills in the teaching learning process.

1.3 Experiences of other countries in incorporating 21CC into mathematics curricula

P21(2021) states that employers and educators around the world generally agree that students entering universities and the world of work after completing school education should have an advanced level of proficiency in mathematics. It posits that one of the most important ways to enable students to achieve mathematical proficiency is to incorporate mathematical content and mathematical practices. P21 further expects that incorporating 21st Century Skills into a core subject like mathematics will make teaching and learning more engaging and ensuring that a greater number of students have an advanced level of understanding and ability in mathematics. Many countries around the world have already incorporated 21CC into the mathematics curricula. For example, Singapore has developed a Framework for 21st Century Competencies and Student Outcomes (See Figure 4) and a Mathematics curriculum framework (Figure 5) which is described below.

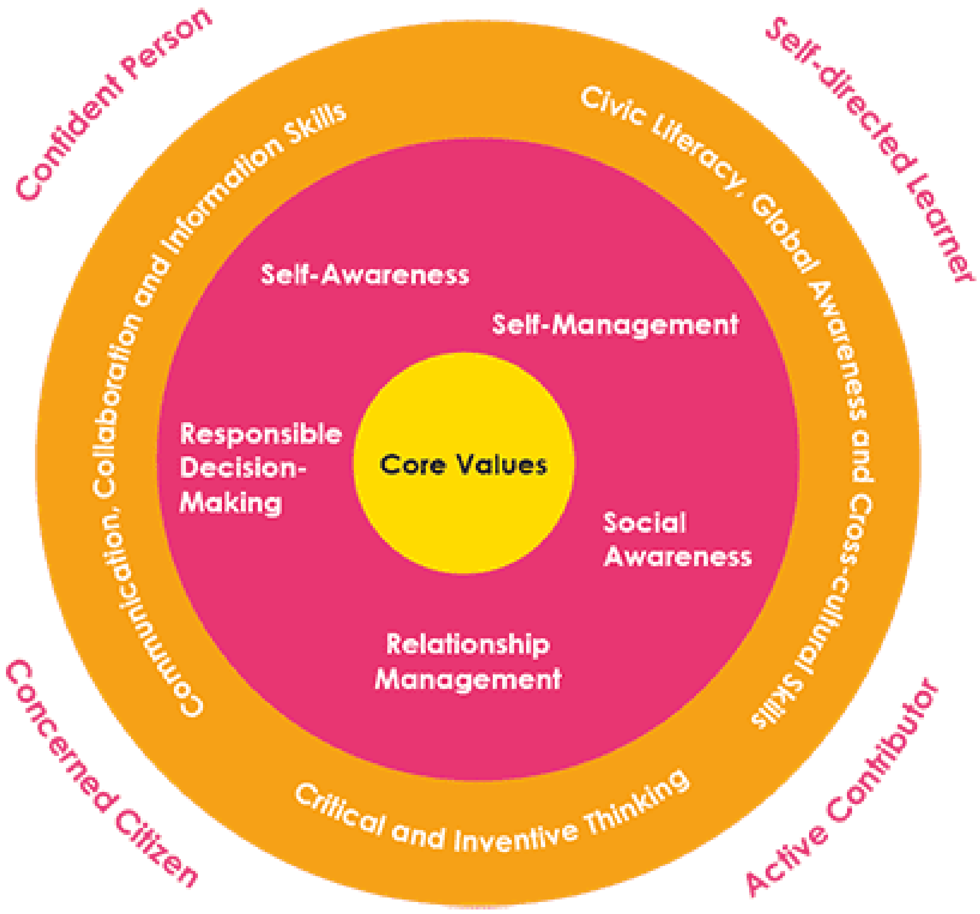


Figure 4: Framework for 21st Century Competencies and Student Outcomes-Singapore
Source: Ministry of Education, Singapore

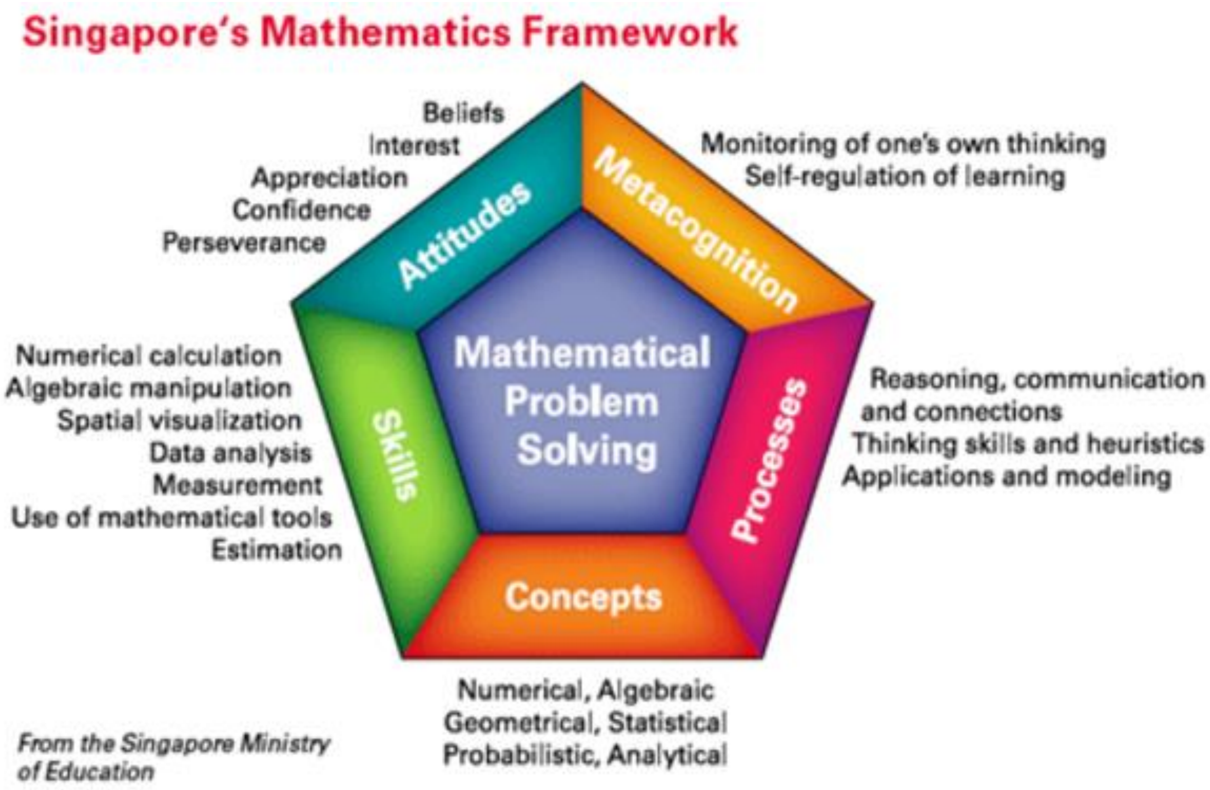


Figure 5: Singapore Mathematics Curriculum Framework
(Source: Ministry of Education, Singapore)

The above framework was first designed in 1990. According to Kaur (2018) every detail of this framework has been carefully thought out and tested over the last 25 or more years. It resembles a house that provides students a 'secure' knowledge in mathematics. For the students to develop a 'secure' mathematics knowledge they need to be confident and capable problem solvers who are equipped to use maths throughout their lives and careers. Therefore, 'Problem solving' is placed at the centre of the house. It must be built carefully. Every house needs a firm foundation. Mathematical concepts gradually build on top of one another. To build higher level concepts safely, you need to build the bottom level concepts firmly. Therefore, teachers lay a firm foundation by not rushing through the syllabus. They use a mastery learning approach. This is the core of 'the mastery approach'. Moreover, 'guided discovery' is preferred to rote learning methods because it allow students to 'discover' mathematics for themselves, albeit with guidance of the teacher.

The walls of the 'house' are the skills and processes that are familiar to maths teachers everywhere. The skills and processes include 21st century competencies as shown in Figure 4. Kaur posits that, the processes that are used in Singapore, have drawn a lot of interest from around the world. For example, “CPA”, or “Concrete-Pictorial-Abstract” is central to the Singapore approach. Good physical and pictorial models are vital to helping students understand abstract concepts. In teaching maths, particular emphasis is on the precise use of mathematical language and the thoughtful use of problem solving heuristics.

The roof on the house comprises two parts. First part of the roof is “Attitudes”: a student with a lot of confidence and enthusiasm will not get far if they don’t have the sufficient knowledge and skills. Equally, a student with all the learning in the world will not take their maths far if they have no confidence or enthusiasm. Having the right attitude is crucial for learning mathematics and a lot of emphasis is on this part of the framework. Singapore has an advantage because it has gradually cultivated a positive attitude to maths across all of society: teachers, parents and, students. According to Kaur (2018) the turnaround started when maths teachers choose to believe that *every child can succeed*. Kaur goes on to say that “This is at the core of the mastery approach and it’s at the core of how we think in Singapore. Teachers need to believe that all students can succeed and reach mastery if they are given right learning experiences and adequate time. When they believe that, it spreads onto the students, and success follows. And then, after a few years, you have a generation of parents who believe it too, and a virtuous circle is created” (Kaur, 2018).

The second part of the roof, and the last part of the framework, is ‘metacognition’. Kaur lucidly explains, how to use the concept in mathematics classrooms:

“Quite rightly, as maths teachers, we spend most of our time asking students to *do* maths. But just occasionally you should try asking them to *think about* maths. You will be rewarded! Get them to keep a journal – not an exercise book, but somewhere to write down thoughts and reflect on their learning. See if they can explain a new concept in words? Or ask a quicker-learning student to explain a concept to one who hasn’t yet grasped it. Both will benefit” (Kaur, 2018).

Kaur further states that Singapore learnt a lot from the UK and the US researchers. It was by listening, researching, and learning from others that the Singapore started to build their system.

Kaur’s fascinating elaboration of the Singapore mathematics curriculum framework reveals that it is developed over a long period of time of about 25 years, and it is based on a sound theoretical foundation, research and practical wisdom. It is an outcome of creativity of Singaporean educationists informed by the systems and research in the UK and the US. Therefore, we can conclude that in Sri Lanka too, it is opportune to develop a sound curriculum framework for mathematics education by learning from more advanced international systems, research, and practical wisdom of our teachers.

1.4 Conclusion

In this chapter we clarified the context of the CAR study, our motivation for this research, and the need to incorporate 21st century competencies into mathematics curricula. In the next chapter we present the methodology that we adopted.

Chapter 2: Methodology

2.0 Introduction:

The discussion on mathematics curricula at the junior secondary level of Sri Lanka in Chapter 1, suggests that the process standards are not properly integrated into the curriculum structure based on content standards. This situation seems to affect the student learning of process skills which are also related to 4Cs and metacognition. Therefore, the issue is how to incorporate process standards, 4Cs and metacognitive skills into the teaching, learning and assessment processes in mathematics classrooms. The issue must be addressed at different levels of administration of the education system, nevertheless we as university researchers and teacher educators are interested in improving classroom practices of teachers to enhance student learning. Research indicates that university researchers collaborate with teachers in classrooms to provide theoretical and research guidance for teachers to conduct Classroom-Based Action Research (CBAR) to improve their practice through CAR (Reil, 2019). Hence, we are interested in addressing the above issue at the classroom level through a CAR approach. In this chapter, first, we elaborate the rationale for of incorporating 4Cs and metacognitive skills to the teaching, learning and assessment processes in mathematics classrooms by using a CAR approach. Then we describe the methodology that we employed in the CAR study.

2.1 The rationale for a CAR approach to incorporating 21CC in mathematics teaching and learning.

Education systems around the world face many challenges in preparing learners for the current global realities which are characterized by rapid economic, social, environmental, and technological change. Education policies and practices need to incorporate the broad range of skills and competencies necessary for learners to succeed in the changing world. They must provide learners with not only the knowledge but also the tools to guide learners to apply that knowledge, and competencies in making decisions to become productive and integral members of society.

Contemporary Sri Lankan educational reforms have focused mainly on top-down, outside-in approaches to changing teachers' classroom practices. "Evidence-based practices" describe "what works in education" and we have witnessed many such practices have been introduced into the education system. "Competency-based curriculum", "Student-centred learning", "E5 model", "school-based assessment", "multilevel teaching", and "Bilingual education" have been introduced at different levels of education over the past few decades. In these reforms, we relied on "what works" in different settings, rather than on teacher judgment and teacher decision making in the classroom. However, current research (Fullan, 2010), reveals that top-down, one-size-fits-all approaches to educational reforms rarely affect the teachers' classroom practices. He posits that professional learning opportunities connected to everyday practice must be provided in a sustained manner over a prolonged period, to bring about real change in teaching.

CBAR provides the teachers, opportunities to engage in "systematic, self-critical enquiry" (Stenhouse, 1985) to solve problems that they face in their classrooms. Following a cycle of inquiry and reflection, action researchers collect and analyse data related to an issue(s) of practice. The focus of action research is on bringing about change in practice, improving student outcomes, and teacher learning (Mills, 2017, Riel, 2019). By situating teachers as scholars and knowledge producers, action research fundamentally changes the culture of contemporary educational reform efforts that de-professionalize teachers (Reil, 2019).

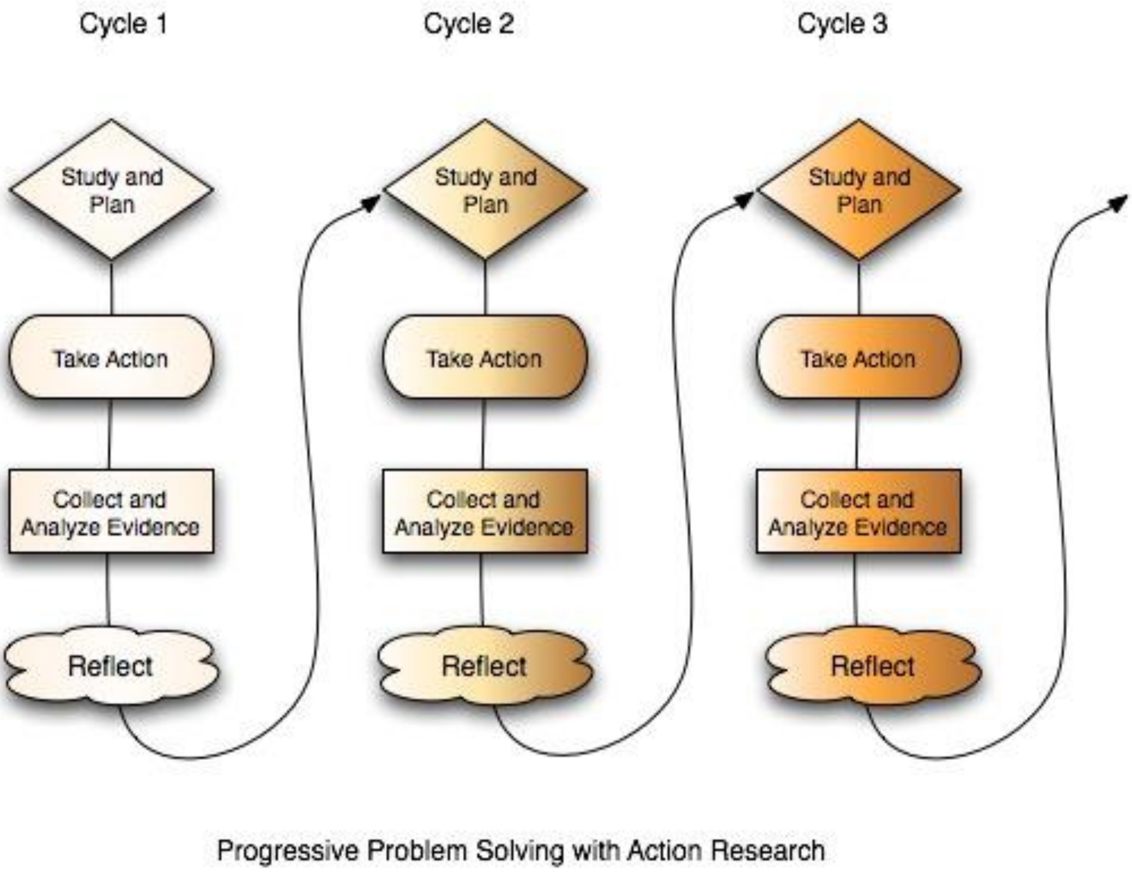


Figure 6: Action research cycle (Riel, 2011)

Figure 6 depicts the nature of action research as a spiral process, consisting of many cycles. According to Riel (2019), each cycle addresses a key question related to the problem identified by the practitioner and therefore, AR is a progressive process of problem-solving. The process of action research starts with a careful analysis of a problem using empirical evidence and developing a plan of action. Then the researcher implements the planned actions while reflecting, observing, and recording the effects of his/her actions. Careful analysis of the data collected through this process leads the action researcher to further reflect upon the success or otherwise of the actions and to plan for the next cycle of actions. Action researcher, then repeats the same steps in the next cycle with the reflectively identified new action and continue the process until the initial problem is solved over time. Hence, the process of action research is progressive problem solving over time. The research takes shape while it is being implemented. Greater understanding from each cycle leads the way to improved practice (Riel and Rowell, 2016).

In our study, teachers' CBAR was integral to the CAR approach that we have used. A team of university researchers assumed the role of initiating and supporting CBAR implemented by a selected group of practitioners for mutual benefits. The outcomes of AR appear at three levels, namely, the personal, organizational, and scholarly levels (Riel, 2019):

1. Personal level: Action research is a socially situated activity, where people are engaged in collective, goal-directed activity. It is a systematic set of methods for interpreting and evaluating one's actions to improve practice. The process of action research involves progressive problem solving, enacted by changing own practice.

2. Organizational level: Action research is about understanding the patterns of interactions that define a social context. Kurt Lewin proposed action research as a method of organizational learning. He claimed that the best way to test understanding an organisation is to try to effect change. The action researcher(s) begin with a 'theory of action' focused on the intentional introduction of change into a social system with assumptions about the outcomes. This theory testing requires careful attention to data, and skills in interpretation and analysis. Theories, such as Activity theory, social network theory, system theories, and tools such as surveys, interviews and focus groups can help the action researchers acquire a deeper understanding of change in social contexts within organizations.

3. Scholarly level: 'The action researcher produces validated findings and assumes a responsibility to share these findings with those in their setting and with the broader research community. Engaging in this dialogue, through writing or presenting at conferences and encouraging and supporting others in the process to do the same, is part of the process of CAR' (Riel, 2019, p.4).

Effective CAR involves a team of practitioners who will work together to plan and coordinate research activities to address an identified problem. CAR has a particular value for practitioners in schools to work towards closing the attainment gap (University of Glasgow (2015). The effectiveness of CAR has been recognized for many years in many settings in which, groups of schools work with each other and key agencies such as universities and educational institutes to address the under-achievement of disadvantaged groups. For instance, the University of Glasgow (2015) claims, 'CAR allows schools and teachers to explore the impact of different methods and approaches that have been tailored to cater for the unique mix of students in their classrooms. Key elements in the collaboration are the careful use of performance and contextual data and access to expert advice and support from local authorities and university researchers' (University of Glasgow, 2015, p. 3).

More specifically, the CAR approach in a school setting has the potential to contribute to:

- improving student learning.
- improving teachers' professional practice.
- wider professional development; and
- overcoming the professional isolation experienced by classroom teachers (Sagor, 1993; University of Glasgow, 2015).

According to the University of Glasgow (2015), CAR is used to improve education and other services in many countries. CAR uses systematic and focused practitioner research to:

- critically examine the current situation of educational phenomena
- identify interventions based on evidence,
- observe the effects of those interventions and
- refine and adapt them as appropriate. (Sagor, 1993; University of Glasgow, 2015)

In CAR, teachers get opportunities to consider together "what's next." When teachers collaboratively develop and test their conceptions and actions, they can better deal with new theories and practices (Schnellert and Butler, 2014). Considering the potential of CAR in bringing about change at the classroom level, and developing teachers' professional learning, we have decided to employ a CAR approach that incorporates a mixed-methods design (Ivankova, 2015) to incorporate 21CC into teaching and learning process of mathematics classrooms. Both quantitative and qualitative methods of data collection were used concurrently to gather contextual information at different phases of the current study.

2.2 The methodology adopted in the CAR

The key question to be answered in this research study is ‘How can we improve student learning in Mathematics at the junior secondary level of education in Sri Lanka by incorporating 21CC into the teaching and learning process in the classroom?’

To address the above key question following sub-questions have been set:

Research questions

1. What is the existing situation of Mathematics education at the junior secondary level in the selected province?
2. What are the key factors affecting teaching, learning and achievements in Mathematics at the provincial and classroom levels?
3. What interventions are necessary at different levels of the education system to improve student learning in mathematics by incorporating 21CC into the teaching and learning process?
4. How effective are the interventions implemented in the study in improving student learning of Mathematics by incorporating 21CC into the teaching and learning process?
5. What are the implications of the findings of this research for policy, practice, and research in mathematics education?

The main purpose of this study is to identify reasons for the existing situation in mathematics education at the provincial level of education and to develop insights for improving teaching and learning through appropriate interventions designed to instill 21CC among students in Mathematics classrooms. To achieve this purpose following objectives have been set.

1. To develop an in-depth understanding of the reasons for poor achievements in Mathematics education at the junior secondary level of education in a selected province.
2. To bring about a positive change in the teaching, learning and assessment practices in the targeted classrooms through a CAR approach.
3. To empower teachers, officers and ISAs who participate in the CAR to identify problems in their practices and to implement appropriate interventions to address those problems to improve their professional knowledge and practices.
4. To generate research-based knowledge for improving policymaking, curriculum designing, resource material production, teacher education and other practices in mathematics education in Sri Lanka.

5. To improve the capacity of university researchers for knowledge creation and contributing to the social and economic development of the country by engaging in collaborative research to improve policy, practice, and further research in the field of education in Sri Lanka.

In this CAR, a team of Sri Lankan university researchers in consultation with a foreign academic attempted to collaborate with Provincial, zonal, and school level practitioners in the Central province of Sri Lanka to improve teaching and learning mathematics at the classroom level for instilling 21CC among students.

2.3 Research process

None of the research team members had relevant practical experiences in incorporating 21CC in teaching and learning in mathematics classrooms. Therefore, we decided to obtain the services of a foreign consultant from the National Institute of Education, Singapore to train a group of resource persons who would conduct workshops for the selected teachers and officers in the planned initial series of workshops. Accordingly, a 4-day workshop was held in October 2019 for the resource persons on the theme of 'incorporating the 21CC for teaching and learning in the mathematics classrooms in Sri Lanka'. A group of 11 teachers, three officers and two ISAs and the university research team participated in the workshop. The workshop was conducted by Professor Berinderjeet Kaur, National Institute of Education Singapore. (See Chapter 3 of this book for the details of the workshop activities and outputs).

The CAR project had been implemented in three phases, namely, the Diagnosis, Planning and implementation and an Evaluation phase as depicted in Figure 6.

Although the process depicted in Figure 7 gives the impression that the implementation of interventions follows one after the other without involving reflection and re-planning, in actual practice interventions were implemented one by one in reflective cycles as indicated by the dashed arrow. Each of the phases represented a reflective cycle of the CAR.

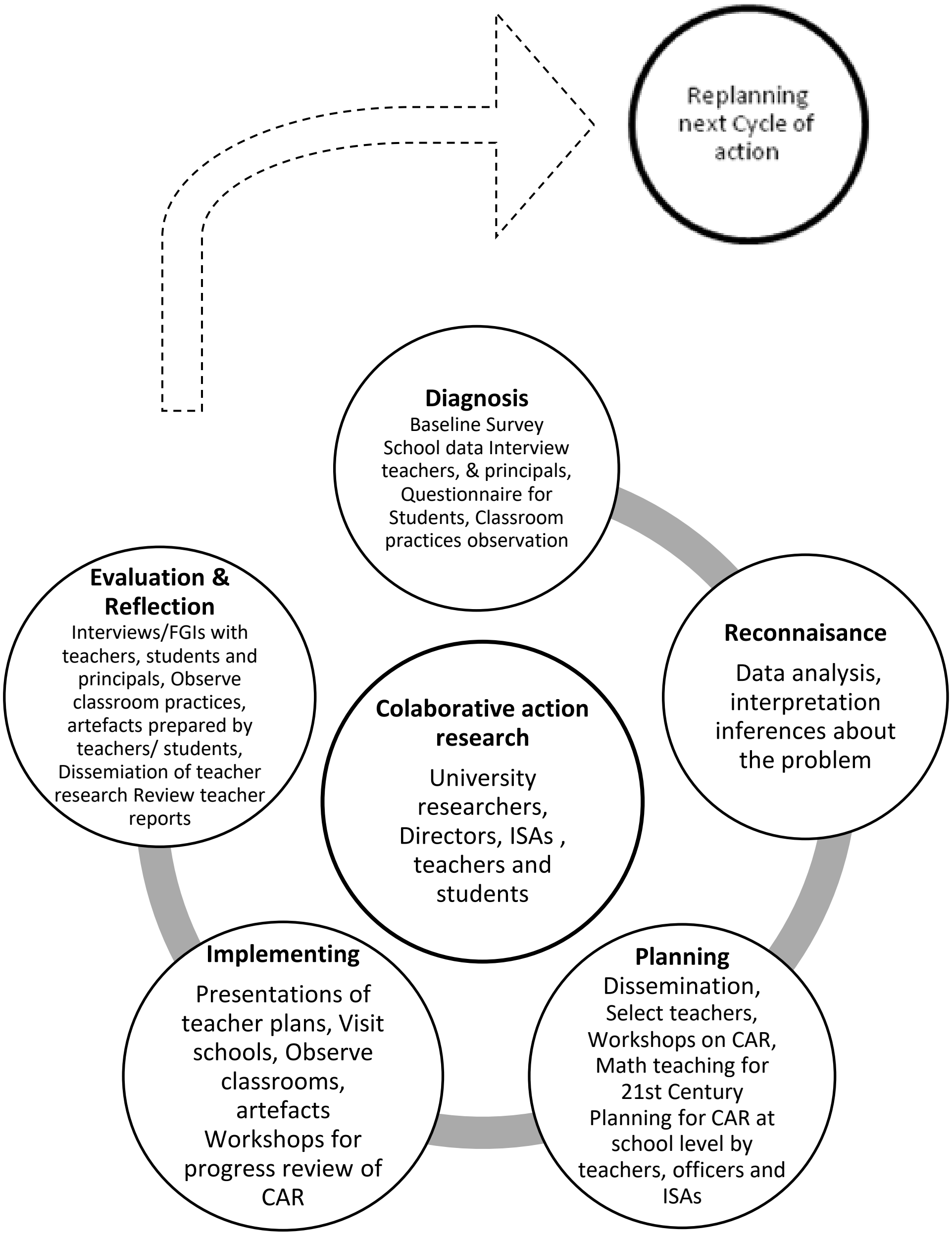


Figure 7: An overview of the Collaborative Action research process with proposed interventions

2.3.1 Diagnosis

The reflective cycle started with an initial step that included both quantitative and qualitative procedures for data collection from 50 schools in the Central province. The second step in the diagnosis was reconnaissance in which the data collected in the initial step had been analysed and interpreted to identify possible causes for the existing situation at the provincial and school levels.

2.3.2 Planning and implementation

In this cycle of the study, we have presented the findings of the diagnosis in a dissemination seminar conducted for the relevant officers, principals and ISAs involved in Mathematics education at the provincial and zonal levels, and mathematics teachers in schools. Twenty five out of 150 participants consented to engage in the CAR at the end of the seminar held in February 2020. The CAR project was to be started by mid-March, however, due to the University closure resulting from the COVID-19 pandemic, the first workshop for the participants had to be postponed indefinitely.

After lifting the lockdown and reopening of the University to the students, in August 2020, we could start the series of initial workshops. Although 25 participants initially consented to participate in the CAR in late February 2020, only 17 participants (2 officers, 1 ISA and 14 teachers) turned up for the first workshop that held in August 2020 after the first lockdown of the country in March.

The key question addressed in this cycle of the CAR was, 'How can we introduce the concepts of 21CC and the ways and means of incorporating those competencies in the teaching-learning process of mathematics classrooms to a group of practitioner-researchers at zonal and classroom levels?' Members of the university research team introduced the concepts of 21CC and Action research in workshops (12 hours in total). A selected set of five resource persons who participated in the 4-day workshop acted as resource persons in the next set of workshops that guided the field assistants (teachers, ISAs, and officers consented to participate in the CAR) on incorporating 21CC in mathematics classrooms.

One of the main purposes of this initial series of workshops was to help the participants to identify a research problem and possible interventions at different levels to improve teaching and learning of mathematics while instilling 21CC among students. Collaborating teachers had been allowed to select their problems for inquiry at the classroom level, while officers and In-service Advisers (ISA) at the zonal level had been allowed to select problems related to their practices at the zonal level. Inputs had been provided in the workshops to facilitate the teachers and other practitioners to plan and implement their action research. Progress review meetings and workshops conducted by the university research team ensured support for planning and implementation of AR at the school level by the participants. In the final workshop of this series of initial workshops, the participants presented their action research proposals and received feedback from the university researchers. We managed to conduct all these initial workshops, face to face with the participants by mid-March 2021. Intermittent lockdowns and school closures/ university closures during late 2020 and the first term of 2021 delayed the completion of this phase until March 2021 which were to be completed by July 2020 according to the original plan. We had to deviate from the original plan to adapt to the new situation created by the pandemic. We have also conducted online progress review meetings to guide the participants in selecting a problem for their CBAR and developing their research proposals, in addition to the initially planned workshops during the above period. Some of the participants dropped out of the project during this period, because of their additional workload in schools, connectivity issues etc. In the final workshop of this series, only 11 participants participated and presented their proposals of CBAR.

Monitoring of the implementation of action research at the classroom and zonal levels was planned to be done by the university research team through progress review workshops and school visits starting from late April 2021. However, the school closure started with the second wave of COVID-19 and the university closure for the outsiders, we had to adopt an alternative strategy for progress review. Instead of conducting face to face progress review workshops, we have decided to conduct online meetings with the teachers to review progress. Teachers also had to use online and other means of teaching during this period. Accordingly, they had to adapt their plans for CBAR to the new situation. Most of the teachers who participated in our project were from rural schools and most of their students (60% or more) lacked digital devices and/or necessary digital infrastructure.

The progress review meetings provided a forum for teachers to interact with university researchers, and other fellow researchers. These workshops facilitated sharing of experiences, review of current actions and planning for the next cycle of actions by different participants. University researchers, directors, ISAs, and teachers have maintained reflective journals where they recorded observations, reflections, relevant information, and decisions. The processes of online teaching by the teachers and online CBAR also hampered by the disruptions that occurred due to trade union action implemented by teacher unions all over the country. So, the teachers had limited time during March-June 2021(except School vacation in April) and late November 2021 to early February 2022 to implement their CBAR.

2.3.3 Evaluation

At the evaluation phase, we have evaluated the outcomes of the action research conducted by the teachers, ISAs and directors using written reports, oral presentations, comments and discussions during progress review meetings, informal interviews, and classroom observations during video productions and the key messages written by the collaborating teacher and officer researchers. At the end of the evaluation, we have planned to conduct two dissemination webinars ; one to share the experiences of the teacher-researchers with other mathematics teachers to receive feedback and the other to present the key findings of the whole study to the National Education Commission, Ministry of Education, NIE and Provincial level Education Authorities. Two books, five conference papers and three journal articles have also been published using the knowledge generated throughout the study.

2.4 Conclusion

In this chapter, we described the methodology adopted in this Collaborative Action Research project. Accordingly, to begin the phase 2 of the study, we have organised a 4-day training of trainers' programme to train a group of teachers, officers, and ISAs on incorporating the 21CC in teaching, learning and assessments in mathematics classrooms. The training programme was held at the Postgraduate Institute of Humanities and Social Sciences (PGIHS) and 16 trainers participated in it. The training programme was conducted by Professor Berinderjeet Kaur, National Institute of Education Singapore. In the next chapter, we present the details of the workshop.

Chapter 3: Incorporating 21st CC in the Mathematics classrooms in Sri Lanka

3.0 Introduction

This chapter gives a detailed description of the training programme conducted by Professor Berinderjeet Kaur of National Institute of Education, Singapore. The activities used in the workshops, important concepts and processes that can be used for incorporating 21st century skills in Sri Lankan mathematics classrooms are summarised in the chapter. The discussions and activities were based on following aspects of mathematics teaching and learning:

- The concept of instructional core,
- Importance of mathematical task,
- Classification of mathematical tasks according to their difficulty levels/Cognitive demand
- Teaching for Understanding
- The importance of classroom discourse
    a. Classroom talk – monologic verses dialogic talk
- Incorporating 21st Century Competences into mathematics teaching and learning
    a. Collaborative problem solving
    b. Reasoning and communication
    c. Metacognition

In this chapter, we briefly describe the above aspects and related key concepts and present a sample of activities used in the workshop. Finally, we present the reflections of the participants on the usefulness of the workshop.

3.1 Workshop for the Resource Persons

The workshop for resource persons on incorporating 21CC into mathematics classroom had been conducted from (02/10/2019 to 05/10/2019). Sixteen teachers, officers and ISAs participated in the workshop together with the university research team. This section presents the intended learning outcomes, workshop contents and related theoretical explanations.

Intended learning outcomes

At the end of the workshop the participants will be able to:

1. Identify 21st century competences and skills for infusion in their mathematics teaching and learning at the junior secondary level.

2. Modify existing textbook tasks for problem solving, collaborative work, reasoning and communication

3. Distinguish monologic and dialogic talk that is part of their classroom discourse and explore strengths and weaknesses of each.

4. Examine teaching and learning actions that nurture self-regulated learning.

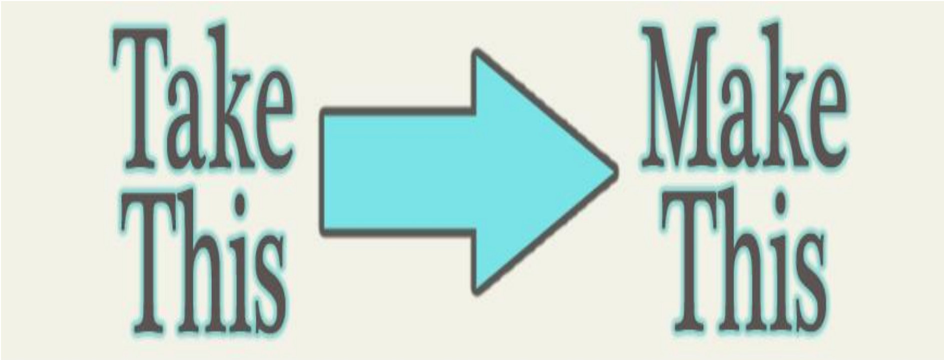
The agenda of 4-day workshop is given in Annex 1.

3.2 Mathematics education in the 21st Century Sri Lanka :What do we aspire to do?

After a discussion on the nature of learners and the competencies required to be successful in life in the 21st century, it is highlighted that the schools are faced with following challenges:

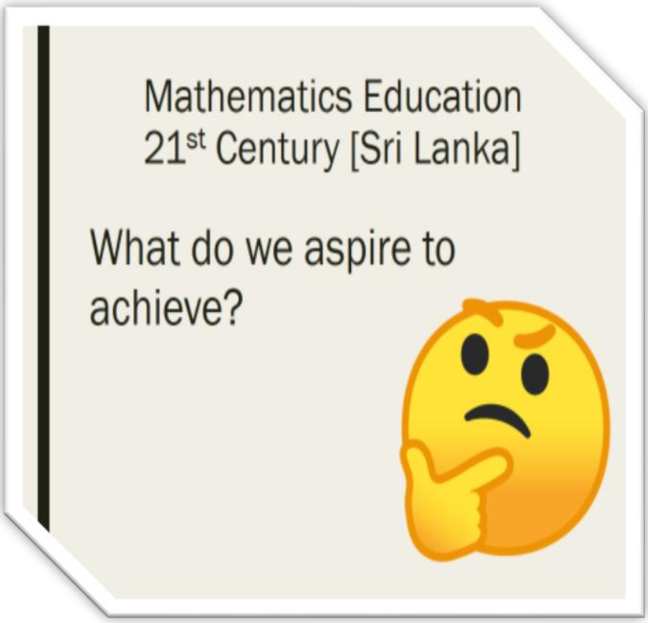
1. Every child graduating from schools should be employable.

2. Since computers or automated machines cannot replace human beings, the children should be developed as thinkers, doers, collaborators, and creators.



The challenge is to transform the existing situation to suit the needs of ever changing socio-economic and technological environments. The learners need to be equipped with appropriate learning and innovation skills or 4Cs. To address the above challenges, Singapore mooted in 1997 'Thinking School Learning Nation (TSLN)' initiative (Goh, 1997). Accordingly, Singapore has developed their own 21st century competency framework and a set of goals for education as elaborated in Chapter 2.

Activity 1: Let's Brainstorm and list our goals collaboratively.



The trainers worked in four groups and prepared a list of goals. During the discussion that followed, the key points were summarised as shown in Table 3.

Table 3: Summary of key points

Tasks that are easy to implement **(Short term goals)**	**Eventual outcomes** **(Long term goals)**
1. Move to student centred learning. 2. Collaborative learning. 3. Activity based learning. 4. Use real world problems. 5. Draw on technology. 6. Create conducive learning environment to develop soft skills and attitudes. 7. Raise awareness among teachers on activities that enhance thinking, collaboration, and creativity among learners. 8. Teachers need to create tasks since textbooks are inadequate.	1. Mathematicians and researchers. 2. Students will become innovative, explorative, and live in a globalised world. 3. Facilitate independent learning using technology. 4. Include parents as co-educators. 5. Workbooks accompanying textbooks. 6. Teacher development. 7. Improve curriculum.
Organisational issues and equipment: • Facilities for Flipped learning • Values and Ethics • Issues beyond the control of educators: Salary/Class size	

The key points were classified as shown in Table 4, by Professor Kaur, to provide a direction for work in the following sessions of the workshop.

Table 4: Tasks, teacher's role, and classroom norms

Tasks	**Classroom Norms**
• Something to do, that the children can't do alone. • Use mathematical language to talk. • Student centred learning • Activity based learning. • Self-learning	• Maintain a safe environment. • Make learning your own.
Teacher's role • Pedagogy for collaboration • Get students to do the task • Use students' work to discuss mathematics • Classroom discourse • Create tasks with differing purposes • Use flipped learning pedagogy- Best way to encourage self-learning. Why did I get this question wrong? Let the students to write a reflection. • Show evidence of success to students. Don't be judgemental. Say, "This is good. Shall we do it differently in the next class?" • Teach students to seek help from others.	**Socio-mathematical norms** • Maintain a safe environment • Respect the teacher • Respect classmates

Important: Teacher must have an aspiration to reach higher goals of innovation. They can use simple techniques to make children think.

They can modify regular textbook questions to make them interesting. For example:

- I'll give you Rs. 500. You are supposed to buy 4 things for your mother. What are the things that you want to buy?

3.3 Instructional core

Instructional core can be defined as the interaction among the three essential elements in an instructional setting as depicted in Figure 8 below.

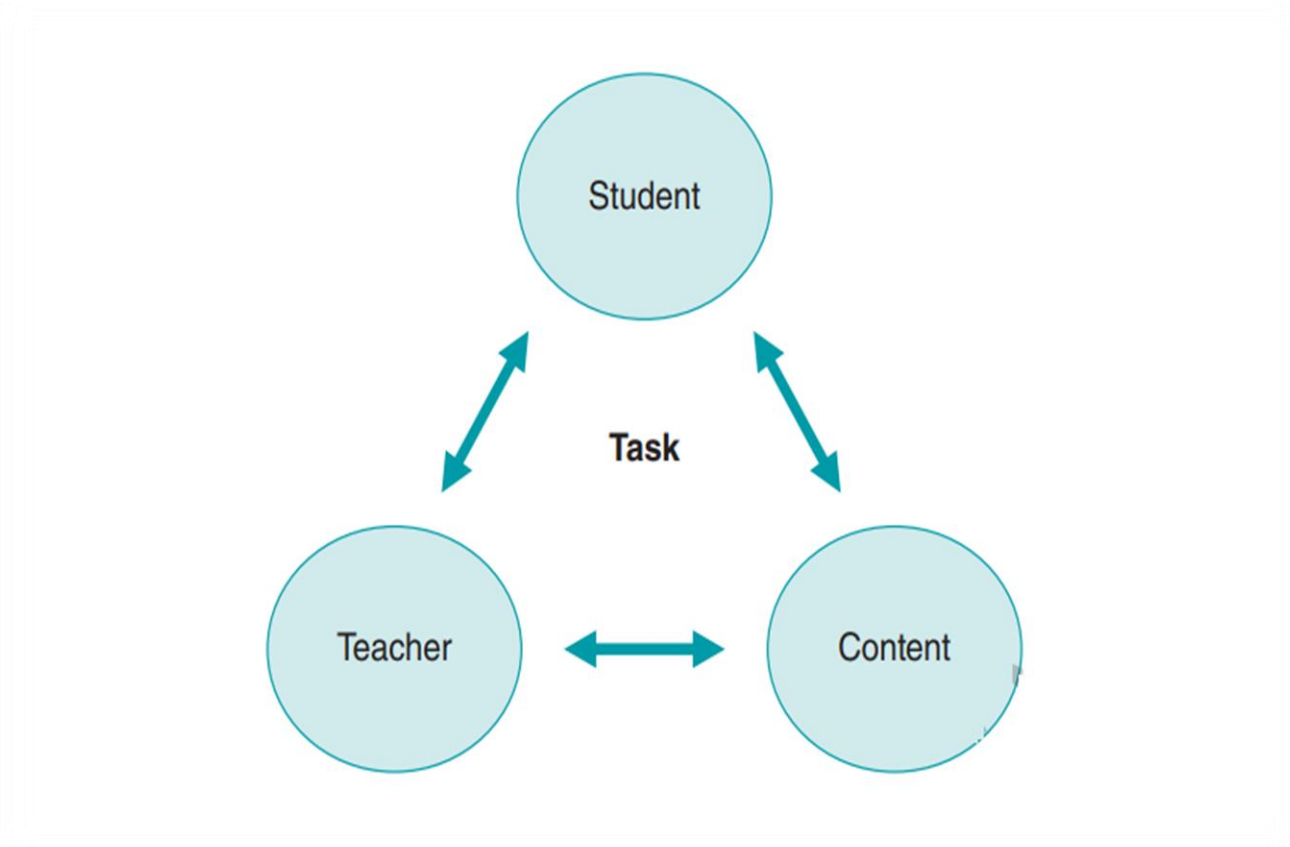


Figure 8: Instructional Core

(Source: Antonetti, J. & Stice, T. (2018). *Powerful task design* (page 8). Corwin.)

Instructional task is at the centre of the instructional core (Doyle 1988). Instructional task is the actual work that students are asked to do. It is *not* what teachers think they are asking the students to do, or what the official curriculum *says* that students are asked to do.

It is argued by many scholars that the choice of task is fundamental to opportunities for student problem solving and reasoning. For example, Anthony and Walshaw (2009), in a meta-evaluation of research, concluded that, "in the mathematics classroom, it is through tasks, more than in any other way, that opportunities to learn are made available to the students" (p. 96).

"Task is the heart of the Instructional Core (IC). When giving a task pause is important- Give the children time to "Think".

3.3.1 Design components of a Task (Antonetti & Garver, 2015)

1. Cognitive demand (the minimal thinking a task will require of the learners)
2. Thinking strategies (the required visible evidence of Personal Response)
3. Engaging qualities (the elements and conditions that elicit energy and enthusiasm)

Task is important to make children engage in learning. Task must be something that elicit excitement, energy, and enthusiasm. They can be arranged from simple to complex level. They can be related to real world context.

Example: If a 9-year-old child joins a class with an average age of 10 yrs, what will happen to the average age of the class?

3.3.2 Levels of cognitive demand of Mathematical Tasks (Stein & Smith, 1998)

Stein and Smith (1998) categorise Mathematical tasks into four levels (0-3), based on the cognitive demand that they exert. Following table describe these levels and the cognitive demands relevant to each level.

Table 5: Levels of cognitive demand of Mathematical Task

Levels of cognitive demand	Characteristics of tasks
Level 0 – [Very Low] **Memorization tasks**	Reproduction of facts, rules, formulae No explanations required
Level 1 – [Low] **Procedural tasks without connections**	Algorithmic in nature Focused on producing correct answers Typical textbook word problems No explanation required
Level 2 - [High] **Procedural tasks with connections**	Algorithmic in nature Has a meaningful / 'real-world' context Explanations required
Level 3 - [Very High] **Problem solving / Doing mathematics**	Non-algorithmic in nature Requires understanding and application of mathematical concepts Has a 'real-world' context / a mathematical structure Explanation required.

Activity 2: Let's examine some mathematical tasks and classify them in the table given below

Task 1.1 What is $1/_2$ of 240?	**Task 1.2** The answer is 120. What fraction of what number could it be?
Task 2.1 Evaluate. • $(\mathbf{2}+\mathbf{4})+\mathbf{6}$ • $\mathbf{2}\times(\mathbf{4}+\mathbf{6})$	**Task 2.2** Explain the meaning of the following expressions and draw pictures representing them. • $(\mathbf{2}+\mathbf{4})+\mathbf{6}$ • $\mathbf{2}\times(\mathbf{4}+\mathbf{6})$
Task 3.1 Evaluate. • $(\mathbf{2}+\boldsymbol{x})+\mathbf{6}$ • $\mathbf{2}\times(\boldsymbol{x}+\mathbf{6})$	**Task 3.2** Explain the meaning of the following expressions and draw pictures representing them. • $(\mathbf{2}+\boldsymbol{x})+\mathbf{6}$ • $\mathbf{2}\times(\boldsymbol{x}+\mathbf{6})$
Task 4.1 State the values of • $\mathbf{Sin}\ \mathbf{30^0}$ • $\mathbf{Cos}\ \mathbf{30^0}$	**Task 4.2** With the help of a diagram explain why Cos 30^0 has the same value as $\mathbf{Sin}\ \mathbf{60^0}$**?**
Task 5.1 Find the length of the hypotenuse of a right-angled triangle with sides $\mathbf{3}\ \boldsymbol{cm}$ and $\mathbf{4}\ \boldsymbol{cm}$?	**Task 5.2** The hypotenuse of a right-angled triangle is 5 cm. What can be the other two sides of the triangle?
Task 6.1 Solve. • $\boldsymbol{x^2}-\mathbf{5}\boldsymbol{x}+\mathbf{6}=\mathbf{0}$	**Task 6.2** What could the equation $\boldsymbol{x^2}-\mathbf{5}\boldsymbol{x}+\mathbf{6}=\mathbf{0}$ represent? What are the values of x and what do they tell you?
Task 7.1 Find the mean, mode and median of the following set of numbers? **2, 3, 3, 3, 5, 8**	**Task 7.2** A set of six numbers have a mean of 4, mode of 3 and median 3. What can the six numbers be?

Source: Kaur, B. (2018). Resource developed for workshop.

Activity 3: Let's classify tasks according to their cognitive demand

Worksheet

Topic: …………………………………. Grade: …………

Select and classify tasks in your text book lessons according to levels of Cognitive Demand (Stein and Smith, 1998).

Level 0 – [Very Low] Memorization tasks Reproduction of facts, rules, formulae No explanations required. __________ __________ __________ __________ __________ __________	Level 1 – [Low] Procedural tasks without connections Algorithmic in nature Focused on producing correct answers Typical textbook word problems No explanation required. __________ __________ __________
Level 2 - [High] Procedural tasks with connections Algorithmic in nature Has a meaningful / 'real-world' context Explanations required __________ __________ __________ __________ __________	Level 3 - [Very High] Problem solving / Doing mathematics Non-algorithmic in nature Requires understanding and application of mathematical concepts Has a 'real-world' context / a mathematical structure Explanation required __________ __________ __________

Source: Kaur, B. (2018). Resource developed for workshop.

3.3.3 Teaching for Understanding (Perkins, 1993): Knowledge vs. understanding

Table 6: Different between knowledge and the understanding

Knowledge	**Understanding**
• The fact. • A boy of coherent facts. • Verifiable claims. • Right or wrong. • I know something to be true. • I respond on cue with what I know.	• The meaning of the facts. • The "theory" that provides coherence and meaning to those facts. • Fallible, in-process theories. • A matter of degree or sophistication. • I understand why it is, what makes it knowledge. • I judge when to and when not to use what I know.

- The fact verses the meaning of facts
  - Examples
    - $12 \times 8 = ?$ Vs what does 12×8 represent?
    - Simplify: $3a + b - a$ Vs simplify $2x + 3y - x$ and explain what this expression may represent?

- A body of coherent facts verses the "theory" that provides coherence and meaning to those facts
  - Examples
    - Circle theorems – disjointed bits verses each leading to the next.
    - trigonometry identities as separate entities verses Pythagoras theorem leading to the three trigonometry identities.

- Verifiable claims verses fallible, in-process theories.
  - Verifiable claims are simple acts of verifying.
    - Example:
      - $x^2 + 2x + 1$ is a perfect square.

    - Fallible, in-process theories are things that can be disproved – not probable in mathematics.

- Right or wrong verses a matter of degree or sophistication
    - Right or wrong?

        - Example:
            - $x + 1 = 3$

              $x = 2$ (Right); $x = 4$ (wrong)

    - Matter of degree or sophisticated

        - Example:
            - To the nearest whole number, estimates, approximations, limits as x tends to zero, infinity, etc.

- I know something to be true verses I understand why it is, what makes it knowledge.
    - Instrumental understanding verses relational understanding (Skemp, 1978)
    - Know the algorithm and can use it verses know how the algorithm came about.
    - Abundant examples in mathematics.

- I respond on cue with what I know verses I judge when to and when not to use what I know.
    - I respond on cue with what I know

        - Example:
            - Use of words such as altogether, left, etc.

    - I judge when to and when not to use what I know.
        - Many examples in mathematics

3.3.4 What is Understanding?

John Dewey (1933) summarized the idea most clearly in his book 'How we think', understanding is the result of facts acquiring meaning for the learner.

> *"To grasp the meaning of a thing, an event, or a situation is to see it in its relations to other things; to see how it operates or functions, what consequences follow from it, what causes it, what uses it can be put to. In contrast, what we have called the brute thing, the thing without meaning to us, is something whose relations are not grasped. The relation of means-consequence is the centre and heart of all understandings" (pp. 137, 146)*

According to Dewy, understanding involves meeting a challenge for thought. When we encounter a mental problem, or puzzling experience with no meaning, we must use judgement, which is based on our own skills and knowledge to solve it.

Bloom (1956) stated that, 'understanding is the ability to marshal skills and facts wisely and appropriately, through effective application, analysis, synthesis, and evaluation.' Completing a mathematical task correctly, therefore, is not, by itself evidence of understanding. It might have been an accident or done by rote.

To understand is to have done it in the right way. The student should be able to explain why a particular skill, approach, or body of knowledge is appropriate or inappropriate in a particular situation.

Undersanding as transferability

To know which fact to use when requires more than another fact. It requires understanding – insight on essentials, purpose, audience, strategy, and tactics. Drill and direct instruction can develop discrete skills and facts into automatically (knowing "by heart"), but they cannot make us truly able.

Understanding is about transfer. In otherwords, to be truly able requires the ability to transfer what we have learned to new and sometimes confusing settings. The ability to transfer our knowledge and skill effectively involves the capacity to take what we know and use it creatively, flexibly, fluently, in different settings or problems. Transferability is not mere plugging in of previously learned knowledge and skill. Transfer involves figuring out which knowledge and skill matters here and often adapting what we know to address the challenge at hand.

Examples:

- To get from his high school to his home, Jamal travels 5.0 miles east and then 4.0 miles north. When Sheila goes to her home from the same high school, she travels 8.0

miles east and 2.0 miles south. What is the measure of the shortest distance, to the nearest tenth of a mile, between Jamal's home and Sheila's home? (The use of the accompanying grid is optional) [New York State Regents Exam]

- What is the distance between the points $(2, 10)$ and $(-4, 2)$ in the XY plane?

  A.6 B. 8 C. 10 D. 14 E. 18

Fact verses transferble knowledge

- $A^2 + B^2 = C^2$
  - a theorem?
  - applicability
    - as a fact/ a rule: therefore only when given right angled triangles
    - transferble knwlege: undersand a context and apply when the situation warrents it as required in the last two examples.

Skill verses transferble knowledge

- Find the LCM/ HCF of a set of given numbers
  - A skill use it for examples such as:
    - find the LCM / HCF of 48 and 60
  - transferble knowledge: undersand a context and apply when the situation warrents it.
    - Example: Green Line buses run every 10 minutes, Red Line buses run every 20 minutes and Purple Line buses run every 35 minutes. After how many minutes will buses from all three Lines next leave the city centre at the same time?

Examples of common misundersandings for some important ideas

- When multiply two numbers, the answer is bigger.
- Multiplication is not repeated addition.
- Fractions when multiplied yeild a smaller answer, and when divided,a larger answer. How can that be?

- Students often see fractions and decimals as separate number systems; learning to see them as alternate means of representing the "same" quantities is the understanding.
- Negative and imaginary numbers are unreal. The undersanding should be that negative and imaginary numbers are no less and no more real than ordinary numbers. Thely exist to provide the symmetry and coninuity needed for essential arithmetic and algebraic laws.

Why learn with undersanding?

- Such learning is generative
- Transferbility of knowledge
    - learn new topics
    - solve new and unfamiliar problems

How undersatnding is developed

Carpener and Lehrer (1999) suggests five forms of mental activity from which mathematical undersanding emerges. They are;

1. constructed relationships
2. extending and applying mathematical knowledge
3. reflecting about expereicnes
4. articulating what one knows
5. making mathematical knowledge one's own

Constructing relationships

- linking informal knowledge to school mathematics
    - use of the counting on method to do subtractions in daily life transactions
    - relating the inventory of a book store to matrix algebra
    - generalizations in real life and symbolic algebra

Extending and applying mathematical knowledge

- knowledge of graphing: extension to graphical represenations in data handiling; solutions of systems of equations.
- Knowledge of simple division: extension to the remainder and factor theorems in algebra
- Use of models to solve algebraic problems by pre-algebra students
- Intuitive knowledge for solving problems leading to mathematical propositions and definitions

Reflecting about experiences – Metacognition

- Reflection involves conscious examination of one's own actions and thoughts
- This is often missing in doing routine tasks where one follows a set of familiar procedures
- Problem solving often engages one in refelction
- Students stand a better chance of acquring this ability if reflction is a part of the knowledge acquisition process
- To be reflective in their learning means that students consciously examine the knowledge they are acqluiring and, in particular, the way it is related both to what they already know and to whatever other knowledge they are aquiring. Learning is not only the acquiring of new concepts and skills but rather the integration of new knowledge with the past - accomodation and assimilation (Piaget).

Articulating what one knows

- The ability to communicate or articulate one's ideas is a benchmark of understanding.
- Articulation involves the communication of one's knowledge either verbally, in writing, or through some other means like pictures, diagrams or models.
- As with reflection, students intially have difficulty articulating their ideas about an unfamilliar topic or task, but by struggling to articulate their ideas especially with means like mathematical symbols or models, students develop the ability to reflect on and articulate their thinking.

Making mathematical knowledge one's own

- Understanding invloves the construction of knowledge by individuals through their own activities so that they develop a personal investment in buliding knowledge.
- They cannot merely perceive their knowledge simply as something that someone else has told them or explained to them; they need to adopt a stance that knowledge is evolving and provisional.
- Othewise they wil see it as someone else's knowledge, which they simply assimilate through listening, watching and practicing.
- The development of students' personal involvement in learning with undersanding is tied to classroom practices in which communication and negotiation of meanings are important facets.

Critical dimensions of classrooms that promote understanding

- Tasks
- Tools
- Normative Practices (Norms)
- Structuring and Applying Knowledge
- Reflection and Articulation
    - Encouraging Reflection
    - A Basis for Articulation

- Classroom Norms
    - Making knowledge one's own

3.3.5 The importance of Classroom Discourse in teaching mathematics

According to Franke et al (2007), how teachers and students talk with one another in the social context of the mathematics classroom, or the mathematical discourse is critical to what 'students learn about' mathematics and about themselves as doers of mathematics.

Through classroom discourse, one can understand how students, the teacher, and subject matter interact in the classroom and how that interaction affects students.

Franke et al (2007) postulate that the development of mathematical understanding in students requires the students to have opportunities in classrooms:

- to present problem solutions,
- make conjectures,
- talk about a variety of mathematical representations,
- explain their solution processes,
- prove why solutions work,
- and make explicit generalizations.

Several research studies have been conducted to better understand the discourse practices that support the development of students' mathematical understanding. These discourse practices are further discussed under classroom talk and dialogic teaching (See sections 3.3.12-3.3.13) in this chapter.

3.3.6 Collaborative problem solving

In Collaborative Problem Solving (CPS), individual students pool their understanding and effort and work together to solve problems. OECD (2017) states that, collaboration has distinct advantages over individual problem solving because it allows for:

- an effective division of labour
- the incorporation of information from multiple perspectives, experiences, and sources of knowledge
- enhanced creativity and quality of solutions stimulated by the ideas of other group members (p. 3).

Collaboration has been defined as a "co-ordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conception of a problem" (Roschelle and Teasley, 1995, p. 70 Cited in OECD, 2017).

In a simpler form of definition PISA 2015 defines CPS competency as follows:

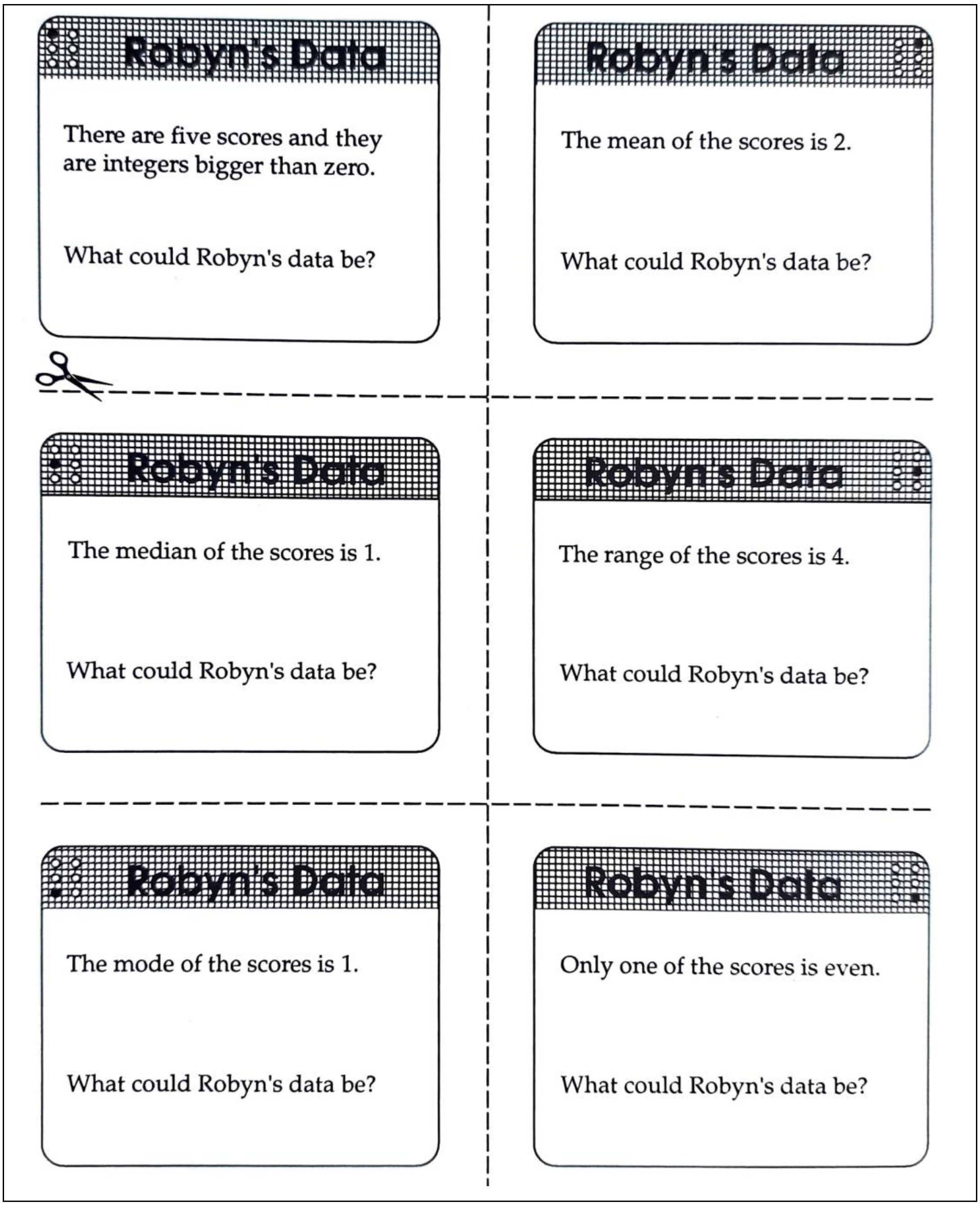
Collaborative problem-solving competency is the capacity of an individual to effectively engage in a process whereby two or more agents attempt to solve a problem by sharing the understanding and effort required to come to a solution and pooling their knowledge, skills, and efforts to reach that solution. (Cited in OECD, 2017, p. 6)

In mathematics classrooms teachers can use CPS to improve individual and collective problem-solving capacities of students.

Activity: 4

Students in groups share set of cards among themselves. Then each member reads aloud the information given in his/her card. They must solve the problem given in the set of cards as a group. The set of cards (Samples 1-8) are taken from Gould, (1993). Sample 1 is given below, and the samples (2-8) are attached as annex-1.

Sample 1 (Gould, 1993, p. 60)



3.3.7 Reasoning and Communication

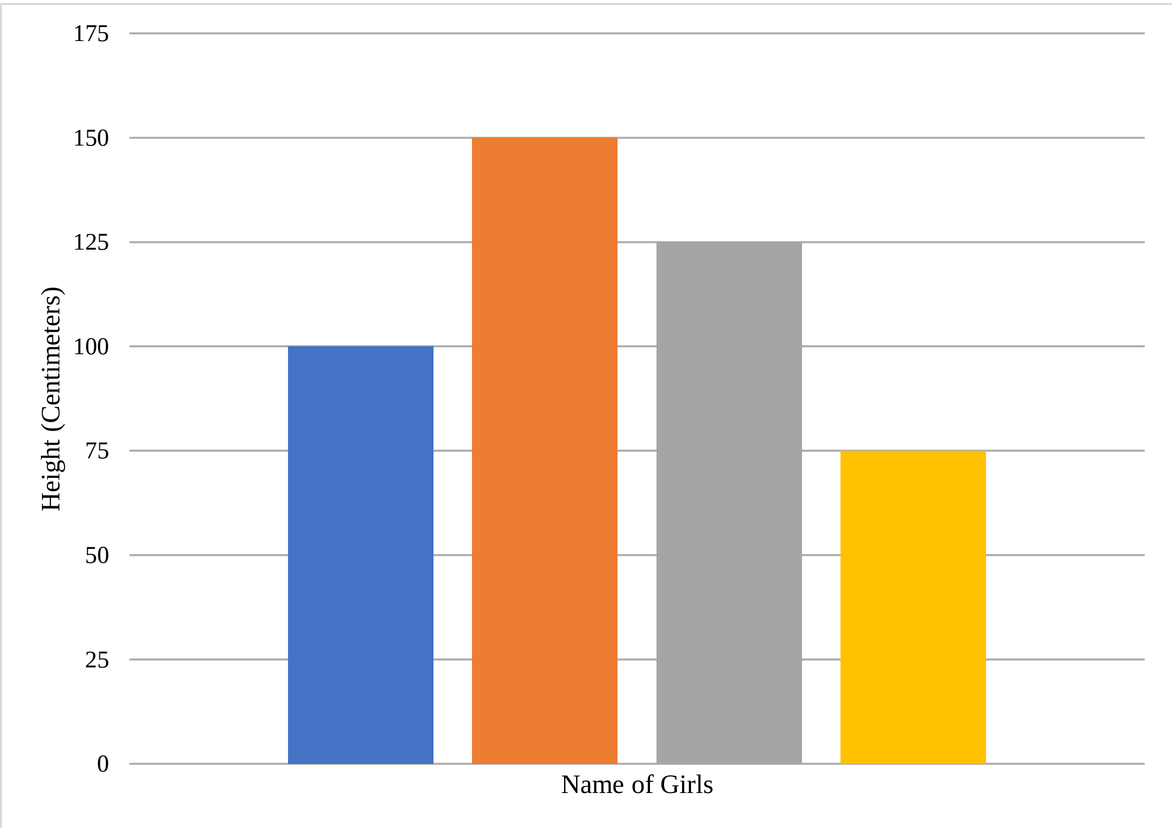
Reasoning is the ability to think, understand and perform of Minions for judgement that are based on facts (Longman, 1987).

Examples of reasoning tasks (Kaur & Yeap, 2009, p. 6-7).

1. Janitha wanted to use her calculator to add $\mathbf{1379}$ and $\mathbf{243}$. She entered $\mathbf{1279 + 243}$ by mistake. Which of these could she do to correct her mistake?

   A. Add 100 B. Add 1 C. Subtract 1 D. Subtract 100

2. The graph shows the height of four girls.



The names are missing from the graph. Debbie is the tallest. Amy is the shortest. Dawn is taller than Sarah. How tall is Sarah?

A. $\mathbf{75\ cm}$ B. $\mathbf{100\ cm}$ C. $\mathbf{125\ cm}$ D. $\mathbf{150\ cm}$

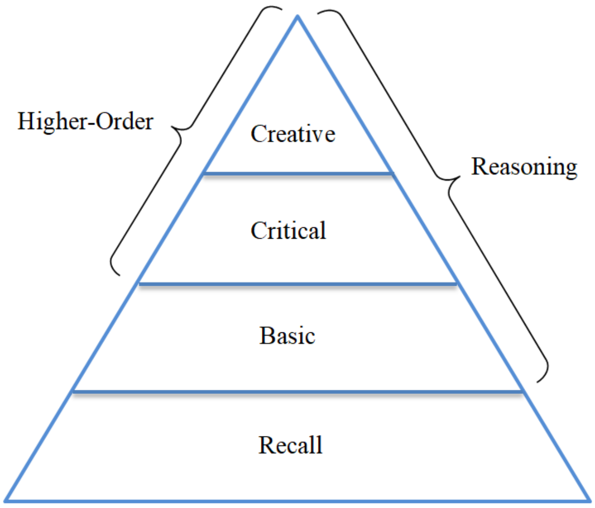


Figure 9 : Hierarchy of Thinking (Krulik & Rudnick, 1993)

Inductive Reasoning **(Kaur & Yeap, 2009, p. 10-12).**

Inductive reasoning is the process of arriving at a conclusion based on a set of observations. It is a method of reasoning particular to general. That is the mental process involved in creating generalisations from observations.

Inductive arguments can include:

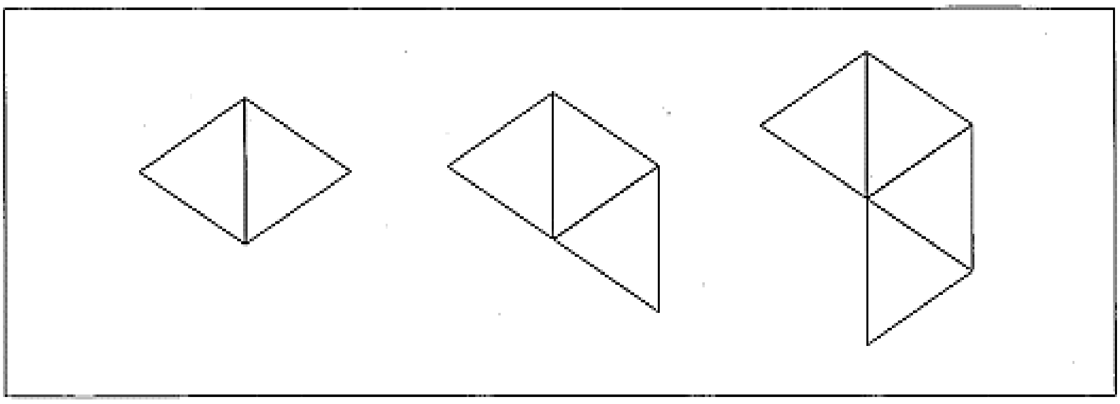
- Part-to-whole: where the whole is assumed to be like individual parts (only bigger).
- Extrapolations: where areas beyond the area of study are assumed to be like the studied area.
- Predictions: where the future is assumed to be like the past.

Examples:

1. Find the missing numbers in this sequence 1265, 1275, 1285, ______, ______, 1315, 1325, …

   By considering the first three terms, one observes that the digit in the tens place increases in a certain way. If a teacher asks pupils to find the 50th term without writing all of them down then the pupils need to derive a general rule.

2. By cutting each shape into as few triangles as possible, find the sum of the angles in the shape.



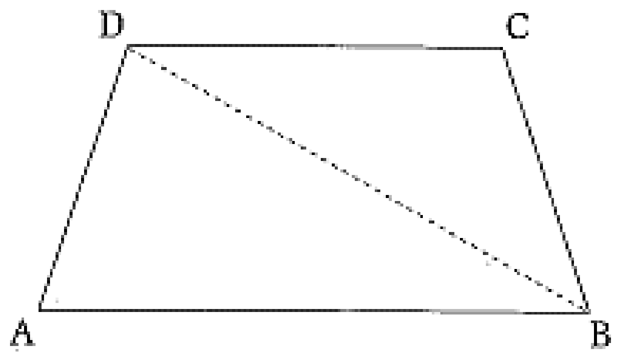
Number of sides	Number of triangles	Sum of angles
4	2	$\mathbf{2 \times 180°}$
5	3	$\mathbf{3 \times 180°}$
6	4	$\mathbf{4 \times 180°}$
7	5	$\mathbf{5 \times 180°}$
8	6	$\mathbf{6 \times 180°}$

a. What is the sum of angles in 50-sided shape?
b. What is the sum of angles in $\boldsymbol{n}$ sided shape?

Deductive reasoning

Deductive reasoning, or deduction, starts with a general case and deduces specific instances. Deduction is used by scientists who take a general scientific law and apply it to a certain case. Deductive reasoning assumes that the basic law from which you are arguing is applicable in all cases. This can let you take a rule and apply it perhaps where it was not really meant to be applied.

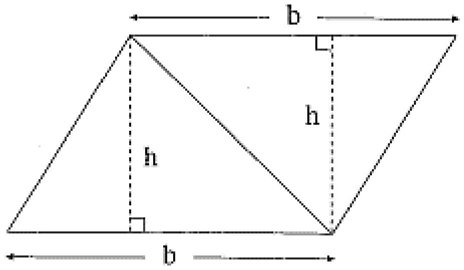
Examples:



1. Find the sum of angles in a trapezium.

Using the general rule that the sum of angles in a triangle is $\mathbf{180°}$, pupils use deductive reasoning to conclude sum of angles in a trapezium is $\mathbf{360°}$.

2. Find the area of a parallelogram.
   Using $\textbf{the area of a triangle} = \frac{1}{2} \times \textbf{base} \times \textbf{perpendicular height}$ and the properties of a parallelogram, pupils can deduce $\textbf{the area of parallelogram} = \textbf{base} \times \textbf{height}$.



Area of parallelogram

$= \frac{1}{2}(\mathbf{b} \times \mathbf{h}) + \frac{1}{2}(\mathbf{b} \times \mathbf{h})$

$= \mathbf{b} \times \mathbf{h}$

$= \textbf{base} \times \textbf{height}$

3.3.8 Modify Textbook Tasks

Activity 5: Modify existing textbook tasks for problem solving, collaborative work, reasoning, and communication.

Here, the participants re-crafted the samples of textbook questions as tasks for problem solving, collaborative work, reasoning and communication.

Strategy 1: What number makes sense (Kaur & Yeap, 2009, pp.14-15)

EXAMPLE 1

A typical textbook question:

A box contained 42 apples. 12 of them were green and the rest were red. Find the ratio of the number of green apples to the number of red apples.

What number makes sense?

Read the problem. Look at the numbers in the box.
Put the numbers in the blanks where you think they fit best.
Read the problem again, do the numbers make sense?

Apples in a box

Mary bought a box of red and green apples.

The box has _______ apples. There are more red apples than green.

There are _______ red apples and _______ green apples.

The ratio of the red apples to the green apples is _______ : _______ .

2	5	12	30	42

See more examples in Annex 3

Strategy 2: What's wrong? (Kaur & Yeap, 2009, pp. 22-23)

EXAMPLE 1

Prize money

John and Henry won a prize of $500 at a Charity Fair. With the money, John bought a bicycle for $140. On their way home they decided to share the prize money equally.

Ali's solution:-
$500 – $140 = $360
$360 ÷ 2 = $180
Each person gets $180

There is something wrong with Ali's solution.

1. Show how you would find the answer to the problem.
2. Explain the mistake in Ali's solution.

See more examples in Annex 3

Strategy 3: What would you do? (Kaur & Yeap, 2009, p. 30)

EXAMPLE 1

Soccer tournament

You are in charge of setting up the school's soccer tournament fixtures.
There are 4 teams competing.
Each team must play once.

1. Create a schedule for the tournament.

2. Explain your reasoning.

See more examples in Annex 3

Strategy 4: What questions can you answer? (Kaur & Yeap, 2009, p 37-38).

EXAMPLE 1

Singapore post

The postage rates for standard sized letters for delivery in Singapore are as follows:

Weight (up to)	Postage
20g	$0.26
40g	$0.32
100g	$0.50
250g	$0.80
500g	$1.00

Write two questions you can answer with the above information.

1. Question 1

____

2. Question 2

____

3. Find the answer to your questions.
Show your work.

____

See more examples in Annex 3

Strategy 5: What's missing? (Kaur & Yeap, 2009, pp. 45-46)

EXAMPLE 1

Donuts

Mary bought 7 boxes of donuts for her class party.
She paid $35 for the 7 boxes.
How much did each donut cost?

Can you find the cost of a donut?

Use the fol1owing prompts to guide you.

(a) What information do you know from the problem?
(b) What else do you need to know to solve the problem?
(c) Pick a number that shows how many donuts might have been in a box. How much would each donut cost?

How much would each donut cost?

Show your workings.

See more examples in Annex 3

Strategy 6: What if? (Kaur & Yeap, 2009, p. 53)

EXAMPLE 1

Cookies and Boxes

Mrs Tan baked 24 cookies.
Each box holds 4 cookies.
At least how many boxes are needed to hold all the cookies?

What if Mrs Tan baked 30 cookies?
What if each box can hold 5 cookies?
What if each box can hold up to 4 cookies?

Generate another 3 "What if" tasks and answer them.
Look out for any interesting observation/pattern.

See more examples in Annex 3

Strategy 7: What's the question if you know the answer? (Kaur & Yeap, 2009, pp. 60-61)

EXAMPLE 1

Red & white chalk

Mr Lee had 3 boxes of red chalk and 8 boxes of white chalk.
Each box contained 5 pieces of chalk.

1. What's the question if the answer is 40 ?

   ____

2. What's the question if the answer is 15 ?

   ____

3. What's the question if the answer is 11 ?

   ____

4. What's the question if the answer is 3 : 8 ?

   ____

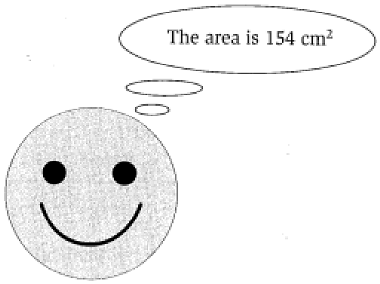
See more examples in Annex 3

Strategy 8: What is the question? (Kaur & Yeap, 2009, pp. 68-69)

EXAMPLE 1

Topic: Area of plane figures

One - Five - Four



1. What could the question be?

   ____

2. My solution is :

   ____

See more examples in Annex 3

3.3.9 Metacognition

John Flavell (1976) originally coined the term metacognition as "one's knowledge concerning one's own cognitive processes and products or anything related to them. Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes (Walt and Maree, 2007). According to Zhang (2009), the term "metacognitive awareness" or "metacognition" is often defined simply as "cognition about cognition" in cognitive psychology and in learning theories in the instructional sciences. Metacognitive awareness enables person to plan, sequence and monitor his or her learning so that the improvements can be seen directly in performances (Kallio *et al.*, 2017).

Metacognition is an important component in the process of effective learning. When a student is more aware with his/her own learning, the student as the learner has more control on the own learning process. As a result, effective development in learning will occur. The metacognitive theory guides the learner to think deeply on his/her own learning process. For the purpose the learner should use metacognitive strategies.

Strategies for Nurturing Metacognition

Adapted from Darling Hammond et al (2001):

- **Predicting outcomes**

  Example of activity: Students are asked to predict who will be the winner when a game is played before they actually play the game to investigate the outcome of the game.

  Students are asked to compare the outcome of the game with their initial prediction. If the outcome is different from their prediction, they will look back at the initial thoughts or possible assumptions/misconceptions made; if the outcome is the same as their prediction, they will think about what are the conditions/information that they used to make the prediction.

- **Evaluating work**

  Example of activity: students ask to review their performance in a test.

Students are to identify what their misconception(s)/error(s) is/are when they have not answered a question correctly and to reflect on and determine how they can avoid making the same misconception/error in future.

- **Questioning by the teacher**

  The teacher asks students:

  - As they work - "Do you understand what you are supposed to do?", "What is the information/condition given in the question that prompt you to take this step?"

  - When they give an answer - "How do you know you are right / wrong?", "Can you justify your answer?", "Is there a better or a more elegant way of obtaining the answer?"

- **Self-assessing**

  Example of activity: Journal writing

  Journaling involves students reflecting on and writing about their learning in mathematics. They can either write based on prompts given by the teacher or simply write freely about their thoughts and feelings on their learning of (a topic in) mathematics.

- **Self-questioning**

  Example of activity: Students ask themselves a series of questions while they work

  Students can use questions to check the understanding and to develop then solve the problem. They can ask themselves a series of questions such that; "What is the question asking for?", "What are the conditions given in the question?", "What are the possible heuristics I may use to solve this problem?" When they ask questions while they work, students can able to direct and clarify their thinking.

- **Selecting strategies**

  Example of activity: Students are asked to decide which strategy, method of substitution or method of elimination, is better to solve a given pair of simultaneous equations.

When students decide which strategy is useful for a given task, they will have to understand the problem to justify the choice of strategy.

- **Use directed or selective thinking**

  Example of activity: (1) Students are required to identify a series of triangles need to solve trigonometric problem. (2) Students to draw a roadmap of steps required to prove a geometrical relationship.

  This process helps students to understand the problem, identify the given information and plan the next/ series of step(s) to take.

- **Using discourse**

  Example of activity: students first work on a problem individually before coming together in a pair/ group to compare their answers. In the pair/ group, each student is to explain how he/she obtains the solution and will have to justify and convince the other(s) of the correct solution.

  This process helps students to concretize their thinking as they can hear their own thinking "visibly". It also so helps students to hear others thinking and identify the gaps in their own thoughts or learn alternative ways of explaining the same concept.

- **Critiquing**

  Example of activity: students are asked to present their solution on the board and the rest of the class will provide (constructive) feedback about the work.

  This process allows students (who are giving feedback) to practice reading and understanding a piece of mathematical work, compare the solution presented with their own to elevate how one solution is "better" than the other. It also allows students (who are receiving feedback) to identify the gaps in their solution and to improve their own thinking process.

- **Revising**

  Example of activity: Students are asked to integrate a series of functions involving logarithms, which day open do not apply the laws of logarithms to simplify the expression before integrating.

Students are shown a better/ more efficient/ more elegant approach (as compared to their own approach) to solve a problem. After learning the alternative approach, students will then make revision to their workings. This process allows students to take note of why the alternative approach is better, check their use of heuristics, and to identify their learning gaps.

3.3.10 Self-Regulated Learning Strategies

Teacher:

- Getting students to set their own learning goals for mathematics at the beginning of each school term/ semester.
- Getting students to plan to revise their work and correct the mistakes.
- Getting students to work with peers to plan for revision and correction of mistakes.
- Getting students to grade their own mathematics work (with the marking scheme/ rubric provided and teach them how to use it).
- Helping students to identify strategies that would help them achieve their learning goals for mathematics.
- Encouraging students to show him/her their plan and review the progress for mathematics.

Students:

- Explaining how to correct an error or a misconception the teacher has put on the board.
- removing their mistakes and identifying possible causes by themselves
- Exploring alternative solution methods for the problem besides one of the teachers as shown on the board.
- Working with peers to review their mistakes, identifying, and justifying possible causes.
- Asking self/ classmate questions to check their understanding.

Kaur, et al., (2019). *Twelve questions on mathematics teaching*. Singapore: National Institute of Education.

3.3.11 Classroom Talk

Teachers need to listen to their students - their questions, ideas, struggles, and strategies of learning, their success, and interactions with peers, their outputs, and views of teaching.

Monologue

Monologue is less satisfactory for struggling, the disengaged, and the confused but powerful for the bright students.

- Ask students to provide answers or solutions (without any explanation) to your questions.
- Ask students to practise a similar problem after you have shown them how to do it on the board.
- Ask students to state/ list what they have learnt at the beginning/ end of the lesson.
- Ask direct questions to simulate students' recall of past knowledge/ check for understanding of concepts being developed in the lesson.
- Provide students with directed guidance (ask close-ended questions) when they face difficulty with the mathematical task they are doing, focusing them on the concept/ skill necessary to do the task.

Dialogue

> ➢ enhances the language of subject
> ➢ empowers the learner actively participate in the construction of knowledge

- Ask students to explain how their solutions or how the answers are obtained.
- Ask students to teach/ explain to another classmate while doing individual assigned seatwork.
- Ask students to explain how to correct an error or a misconception that you have put on the board.
- Ask students to justify why their answer to a problem is different from the one you have put on the board.
- Ask students to defend and explain to classmate(s) why their approach/method to solve a problem is better (more efficient or more elegant).
- Ask students to work with us to review their mistakes, identify and justify possible causes.

3.3.12 Dialogic Teaching (Alexander, 2008)

- **What is dialogic teaching?**

Dialogic teaching harnesses the power of talk to simulate and extend pupils' thinking and advance their learning and understanding.

Dialogic teaching pays as much attention to the teacher's talk as to the pupils.

Dialogic teaching is ground in research on the relationship between language, learning, thinking, and understanding, and in observational evidence on what makes a good learning and teaching.

- **Is it a method of teaching?**

No. Dialogic teaching is not a single set method of teaching.

Dialogic teaching is an approach and a professional outlook rather than a specific method. It is concerned not only with the techniques we use but also the classroom relationships we foster, the balance of power between teacher and taught and the way we conceive of knowledge.

- **What does it look like in practice?**

In a dialogic classrooms' children don't just provide brief factual answers to 'test' or 'recall' questions, or merely spot the answer which they think the teacher wants to hear. Instead, they learn and are encouraged to:

- *narrate*
- *explain*
- *analyse*
- *speculate*
- *imagine*
- *explore*
- *evaluate*
- *discuss*
- *argue*
- *justify*
- *ask questions of their own*

In learning, as in life, all these forms of talk are necessary. To facilitate the different kinds of learning talk, children in dialogic classrooms also:

- *Listen*
- *Think about*
- *Give others time to think*
- *Respect alternative viewpoints*

Many of the teachers in the dialogic teaching development projects have negotiated ground rules for talk along the lines above, and there are frequently reviewed with pupils.

In dialogic classrooms teachers consciously use *discussion and scaffold dialogue*, as well as the other kinds of teacher talk.

- **What do you mean by "Scaffolded dialogue"?**

Discussion entails the open exchange of views and information to explore issues, test ideas and tackle problems. It can be led by one person (the teacher of pupil), or it can be undertaken by the group collectively. **Scaffolded dialogue involves:**

- *Interactions* which encourage children to think, and to think in different ways
- *Questions* which require much more than simple recall
- *Answers* which are followed up and build on rather than merely received
- *Feedback* which informs, and leads thinking forward, and provides encouragement
- *Contributions* which are extended rather than fragmented
- *Exchanges* which chain together into coherent and deepening lines of enquiry
- *Classroom organisation, climate and relationships* which make all this possible

- **Do you have to organise the class in a particular way for dialogic teaching?**

In dialogic classrooms teachers exploit the potential of five main ways of organising interaction to maximise the prospects for dialogue:

- *Whole class teaching*
- *Group work (teacher-led)*
- *Group work (pupil-led)*
- *One-to-one (teacher and pupil)*
- *One-to-one (pupil pairs)*

Again, all of these have their place: no one form of interaction on its own will suffice for the varied purposes, content, and contexts of a modern curriculum.

- **What are the principles of dialogic teaching?**

Whatever kinds of teaching and learning talk are on offer, and however the interaction is organised, teaching is more likely to be dialogic if it is:

Collective
Participants address learning tasks together

Reciprocal
Participants listen to each other, share ideas, and consider alternative viewpoints

Supportive
Pupils express their ideas freely, without fear of embarrassment over 'wrong' answers, and they help each other to reach common understandings

Cumulative
Participants build on answers and other oral contributions and chain them into coherent lines of thinking and understanding

Purposeful
Classroom talk, though open and dialogic, is also planned and structured with specific learning goals in view.

3.4 Template for designing classroom tasks to make students, 21st century skills champions

You want to be a 21st century skills champion

1. Grade:

2. Topic:

3. Regular textbook task:

4. A new task for 21st Century skills

Questions/Prompts — 21st Century skills

Students will:	Example:
------------------------------------ ------------------------------------ ------------------------------------	Collaborative problem solving
Metacognition: (Quiet time for the students-Ask students to close their eyes and think) What did my teacher teach today? What do I think about it?	

TWO FINAL THOUGHTS

'If an answer does not give rise to a new question from itself, it falls out of the dialogue' (Mikhail Bakthin).

'What ultimately counts are the extent to which teaching requires pupils to think, ok not just report someone else's thinking' (adapted from Marin Nystrand et al (1997))

(See Annex 4 – Reflections of the participants)

3.5 Conclusion

In this chapter we have attempted to reconstruct the workshop using the written records of the workshop and other artefacts used by Prof. Kaur. The workshop highlighted the importance of mathematical tasks and discourse used by teachers in classrooms for developing thinking skills, understandings, collaborative problem solving, reasoning and communication, representation of mathematical ideas in visual forms and metacognition among learners. Moreover, the participants had opportunities to critically look at textbook tasks and to modify them to facilitate the development of students' higher order thinking by designing tasks with higher levels of cognitive demand and also to develop 4Cs. Reflections of the participants clearly indicated that most of them grasped the meaning of these practices and the fact that they want to make use of that knowledge in their classrooms and to sharing such knowledge with other teachers in their different capacities. The workshop has been successful in achieving its objectives. Now the challenge is to make use of trainers' knowledge for training other teachers and officers in the above aspects and ultimately developing the above skills among mathematics learners at the junior secondary level. The next chapter presents the process adopted in this endeavour while a separate volume gives a descriptive account of the CBAR projects undertaken by individual teachers, officers, and ISAs.

Chapter 4: Implementation of CAR: Cycles of inquiry

4.0 Introduction

Action research is a collaborative process, which involves people in a social context such as a workplace, organisation, or a community setting. It is a systematic and reflective study of one's actions and their effects, in a social context. The purpose of action research is to develop a deeper understanding and insights about a social issue by implementing deliberate actions, reflecting upon the consequences of those actions, and learning from them. Moreover, as a form of research, action research implies a commitment to data sharing and knowledge construction (Reil, 2019). Therefore, in this final chapter our purpose is to describe the process of implementation, report our reflections about our actions and their consequences, understandings and insights developed on our own practices and how we shared our understandings with relevant others in the process.

4.1 Process of Implementation

CAR also follows a spiral process depicted in Figure 10. It involves deliberately planned actions, Collection, analysis, and evaluation of evidence, reflections and knowledge construction and sharing with important others. In our study, we followed a similar spiral process, as depicted in Figure 10.

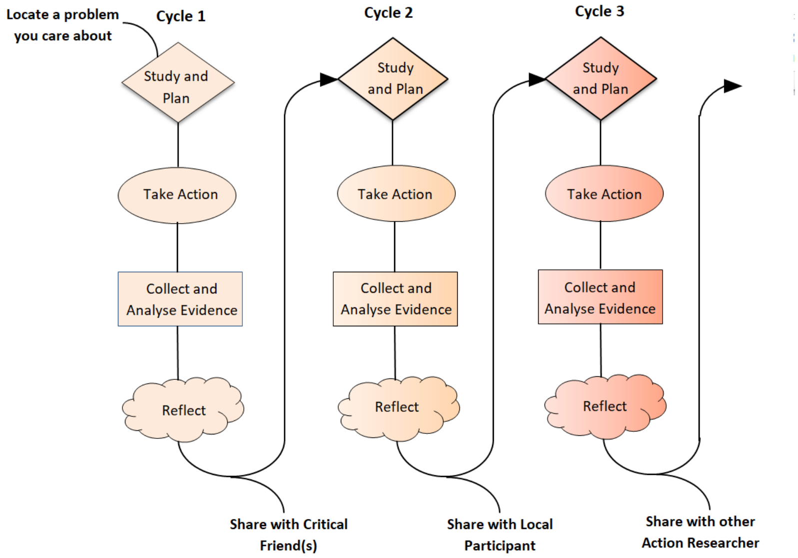


Figure 10: Action research as a spiral process of data sharing and knowledge construction (Source: Reil, 2019).

We shared our understandings in the process, first, as critical friends with other collaborators and then with academic communities by participating in conferences and through our publications.

According to Berieter and Scardamalia (1993), action research provides practitioners a path of learning from and through one's practice using a series of reflective cycles that facilitate the development of progressive problem solving. In the current study, we have attempted to learn reflectively through our actions aimed at improving student and teacher learning in mathematics classrooms. Accordingly, we have identified three distinct reflective cycles, where we addressed three main questions:

1. How can we introduce 21CC into mathematics teaching and learning in the Junior secondary level classrooms through CBAR by teachers?
2. How can we facilitate teacher professional learning through CBAR?
3. How effective is the CAR and the CBAR implemented by teachers/ISAs/and officers and how can we share our understandings with important others?

The current chapter addresses the first two questions and Chapter five will address the third question. Three action cycles have been used to address each of the above questions.

4.2 The first cycle of CAR:

<u>How can we introduce 21CC into mathematics teaching and learning in the Junior secondary level classrooms through CBAR by teachers?</u>

The first cycle of CAR started with conducting a 4-day workshop to train the resource persons on incorporating 21CC in teaching and learning in mathematics classrooms at the junior secondary level. Twenty participants, including the university research team, three education officers, two ISAs and 11 mathematics teachers participated in the workshop. The workshop was conducted by Prof. B. Kaur of the National Institute of Education, Nangyang Technological University in Singapore. Prof. Kaur is a Professor of Mathematics education with long years of experience in both teaching and researching mathematics education at different levels. Our purpose of inviting her was to benefit from her scholarly and practical experiences of training mathematics teachers, research, and consultancies in mathematics education.

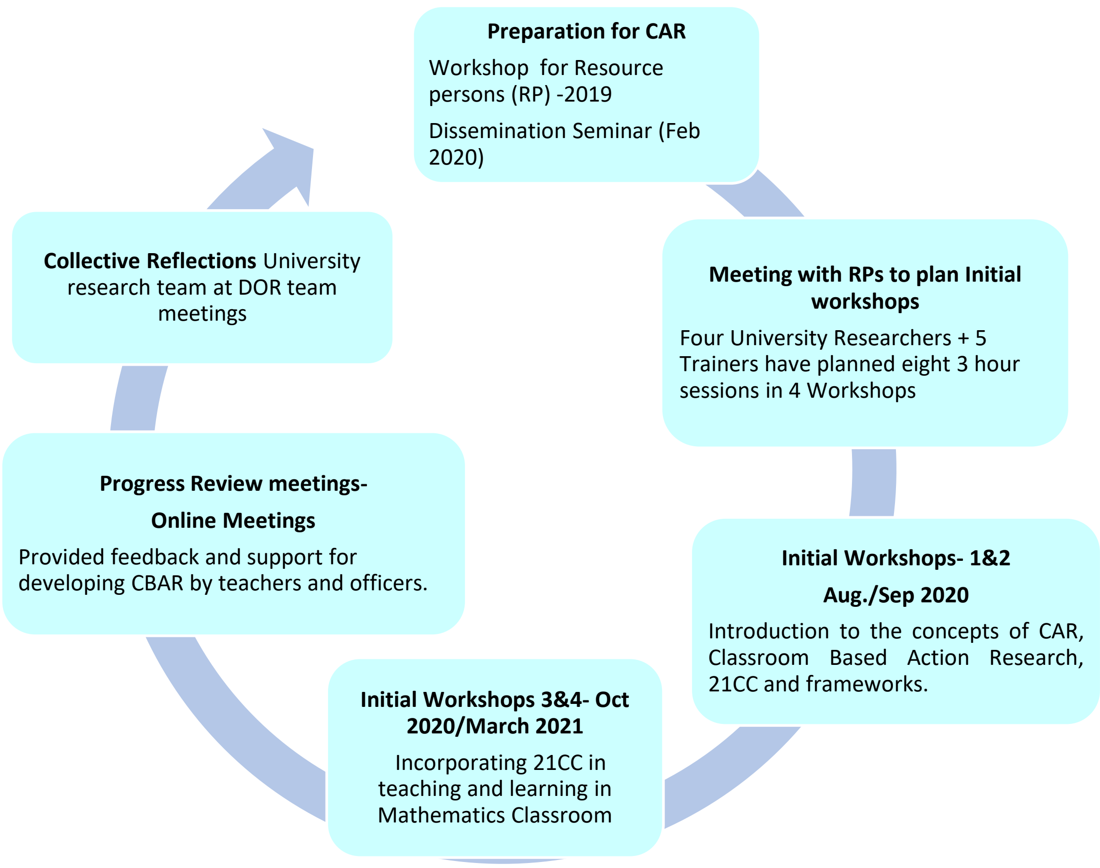


Figure 11: First cycle: How can we introduce 21CC into mathematics teaching and learning in the Junior secondary level classrooms through CBAR by teachers?

In the workshop Professor Kaur, first, enlightened us on the vision of education in Singapore (Thinking School, Learning Nation-TSLN), incorporation of 21CC in education, and especially into mathematics curricula at school level. The concept of 'thinking school and learning nation' emphasises, improving thinking skills and independent learning skills among students. People view 21st century differently and expect every child graduating from school should be employable. Machines cannot think independently, and therefore, cannot replace human beings. Education needs to develop thinking abilities of students to create new ideas, designs, processes, and materials. It should also inculcate appropriate values among learners and build character and soft skills. In Singapore, children sing Family song, National Anthem and make Singapore pledge every morning in school. Prof. Kaur recited the Singapore pledge herself. While listening to her I had the feeling that it is so practical to use such a pledge in schools to develop patriotic attitudes and values among future citizens

In the first activity of the workshop, Prof. Kaur asked the Sri Lankan participants to indicate their expectations for the 21st Century mathematics education in groups. The group responses were later summarised into a Table indicating short term and long-term goals of mathematics education of the 21st century, Sri Lanka. The outcome of this exercise laid a firm foundation for the rest of the workshop. Key concepts and activities introduced in the workshop seemed to help the participants to look at mathematics teaching from a novel perspective. Teacher can modify the tasks given in textbooks and Teachers Instructional Manuals (TIM) to teach students higher order cognitive skills and 21CC. The participants learnt in the workshops many ways of transforming textbook tasks into more cognitively demanding tasks that facilitate students' higher order thinking and achieving 21CC. Prof. Kaur also emphasised the importance of mathematical discourse in the classroom and learning mathematics with understanding rather than rote learning. The effects of what the trainers learned in the workshop and they later shared with the collaborating teacher researchers could be observed in CBAR implemented by the latter group. Moreover, those effects seem to have influenced the research proposals of MPhil students, designed and implemented later in the project.

The other important action implemented in the first step of the first cycle of CAR was the dissemination seminar, where we presented the key findings of the survey research study conducted in Phase 1 to a group of about 150 participants consisting of education officers at the Provincial and Zonal levels, ISAs, Principals, and mathematics teachers. The main purpose of dissemination seminar was to sensitise the participants in general to the issues related to mathematics teaching, learning and students' achievements in the province and to recruit a group of participants for the CAR. Participants' feedback comments indicated that the dissemination seminar was a rare experience for them that discussed the issues related to teaching and learning of mathematics. Twenty five participants gave their consent to participate in the CAR at the end of the seminar. It was quite disheartening to see that none of the officers including the participants of our training of trainers workshop, who were above the rank of Assistant directors consenting to participate in the CAR. Their reason for inability to participate was either the work overload or personal problems. However, the teachers were more enthusiastic than the officers in participating in the CAR.

The next step in the first cycle was conducting initial workshops to facilitate the teachers to identifying a research problem and writing a proposal for their CBAR studies. As depicted in Figure 11 four initial workshops were conducted. Most of the teacher participants have selected topics related to improving students' problem solving abilities and higher levels of cognitive skills. Table 7 indicates tentative titles of the research proposals developed by the participants.

Table 7: CBAR proposals developed by practitioners

Name	**Proposed Titles**
Ms Thishani Bandaranayake	How can I help students to change their attitude that 'Mathematics is a difficult subject'?
Ms Darshani Herath	How can I help students to improve problem solving ability by developing collaboration and communication skills?
Ms Kumudu Ariyarathne	How can I help students to improve their ability to solve word problems?
Ms Dhammi Polgaspitiya	How can I motivate mathematics teachers to use ICT for improving teaching and learning Geometry
Ms Niroshi Ekanayake	How can I improve students' active participation in mathematics classroom?
Ms Lochanie Adhikaram	How can I help students to engage in higher order thinking (Analysis/ synthesis) in mathematics classroom?

Ms Sandamali Mahakumbura	How can I help students to improve their problem-solving skills in Mathematics?
Ms Bhagya Gallala	How can I motivate teachers to use activity-based learning for improving mathematics achievements?
Ms Ayomi Wijesuriya	How can I use structured activities to improve students' problem-solving ability?
Ms Punsara Nilupuli	How can I make learning mathematics attractive to the students with language problems (Tamil students in a Sinhala medium classroom)
Mr. Kavinda Wijethunga	How can I help students to improve students' problem-solving using metacognition?

Although we initially planned the first cycle of CAR to be completed in the final quarter of 2020, the completion of the cycle was delayed until March 2021 because of lockdowns and school closures during the first wave of COVID-19 Pandemic. After the third workshop that we conducted in late September 2020 the country went into a lockdown again, and in January 2021 we have conducted an online meeting with the participants to provide further guidance on research problem identification and proposal development. Some of our teacher researchers and officers dropped out of the CAR project during this period due to various reasons such as connectivity problems, the lack of digital infrastructure, and teachers' work overload due to prolonged school closures. Despite these setbacks we decided to continue the project with the remaining 8 participants.In Cycle 2 and 3 respectively, we had opportunities to share the participants' experiences in implementing CBAR and to act as critical friends to them in the process, especially through the progress review meetings.

4.3 The Second Cycle of CAR:

<u>How can we facilitate teacher professional learning through CBAR?</u>

Figure 12: Second Cycle: How can we facilitate teacher professional learning through CBAR?

Initially, we have planned to conduct progress review meetings with the participants and school visits by the research team in this cycle of the CAR to support implementation of CBAR. However, because of the school closure at the end of April 2021, it made the teachers to use online mode or social media and other measures for teaching. Country lockdown in August 2021 and the prolonged trade union action by teacher unions, also affected the implementation of CBAR until the reopening of schools for JS students in late November 2021. We could not visit schools, to observe the implementation of CBAR by the participants and provide feedback. Therefore, we decided to use online meetings to review progress of CBAR. Our collaborating teachers and officers also had to use online or blended learning measures to conduct their CBAR. (The details of CBAR by individual researchers are presented in another volume).

In the online meetings each participant presented their progress of CBAR using PowerPoint. These meetings helped us to understand strategies used by them, their thoughts, and the problems they faced in implementing their planned actions. An extract from the transcription of the first progress review meeting in the second cycle is given bellow.

Lochani: I have planned to use 10 lessons in the first term to improve students' achievements in solving problems with understanding. I was teaching 'Perimeter' to Grade 7 students.

After 2 weeks, I gave them a paper send by the zonal office and compared their marks. To my dismay, I found that their scores decreased. There were 8 such students. The children said, 'sums were difficult, madam'. I was saddened.

Then the school closed. We were asked to teach online. I have created WhatsApp groups and continued teaching with difficulties.

Subhashinie: What type of difficulties?

Lochani: Some children don't have devices. To participate in online lessons, some others go to a neighbour's house.

This time I did not give difficult sums for them to do. I gave them sums that they prefer to do. If I give more complicated problems, they might not answer. I did 4 online lessons. Only 16 out of 31 send the answers. Others did't have devices. For one student I gave a phone that we had at home.

Subhashinie: So, what are you going to do next?

Lochani: In school, when I gave them more difficult sums, First, I helped them to understand the problem and then allowed them to do the sum. But, now with online, I can't do the same. I can't check what they do or how they do it. So, now I give only simple sums.

Subhashinie: What If, you add at least one little more complicated question? And ask them to try it. You may tell them that you just want to see who will succeed in doing it in the next class or say that you'll give a reward to the ones who attempt to do that? Or you may think about another creative way of getting them motivated to try more difficult problems.

I enjoyed this type of dialogues with the participants in progress review meetings. They help us to get a glimpse of what really happens in their mathematics classrooms especially the difficulties and dilemmas faced by teachers and students. They allowed me to think about an alternative way of addressing such issues and suggesting a strategy for the teacher. It was quite challenging at times. Here, I knew my suggestion is not the best solution for the problem, but

I wanted them to think about an alternative and try it out. I did not want them to follow my suggestion blindly.

Some of the classroom issues such as lack of devices and Wi-Fi were beyond the control of teachers and the research team. Developing self-learning materials for the 'digital have nots' would have been one solution. However, due to time and other resource constraints of the teachers we couldn't implement it.

The conversations we had with teachers in the online progress review meetings also revealed the teacher identities, beliefs and expectations that affect their classroom practices.

Kavinda: I tried to improve students' problem solving abilities using metacognitive strategies. Children are eager to get a numerical answer. Most of them do not try to understand the problem. I thought I can teach them problem solving stages of Polya.

He explained, how he used some activities to guide the students through Polya's model of problem solving. We felt that it was an experimental design rather than action research. Reflection in and on action were missing to some extent. He observed that some students quickly grasped his ideas but many of them were struggling. He appeared frustrated about the slow progress of some students as implicated in the above quotation. We suggested him to reflect on students' difficulties in diagramming a problem or giving opportunities for the students to explaining how they did the sum etc, and what can he do to help them, before implementing the subsequent activities. Prasad suggested him to write down expected outcome of each action that he implements.

In each meeting we have listened to each participant and gave feedback in the form of a complimentary remark, suggestion, or a question for further reflection. We tried to play a role of a critical friend in these progress review meetings. Feedback we received in these meetings also led us to plan our strategies for the next step. Since we were unable to conduct face to face sessions with the participants during the pandemic, we decided to conduct online workshops to support the CBAR implementation. Our interactions with the participants in the previously held online meetings and our individual and collective reflections guided our online workshops. At the research team meetings, we collectively reflected upon the outcomes of the online meetings and decided to conduct a series of six online workshops on the themes depicted in Figure 13.

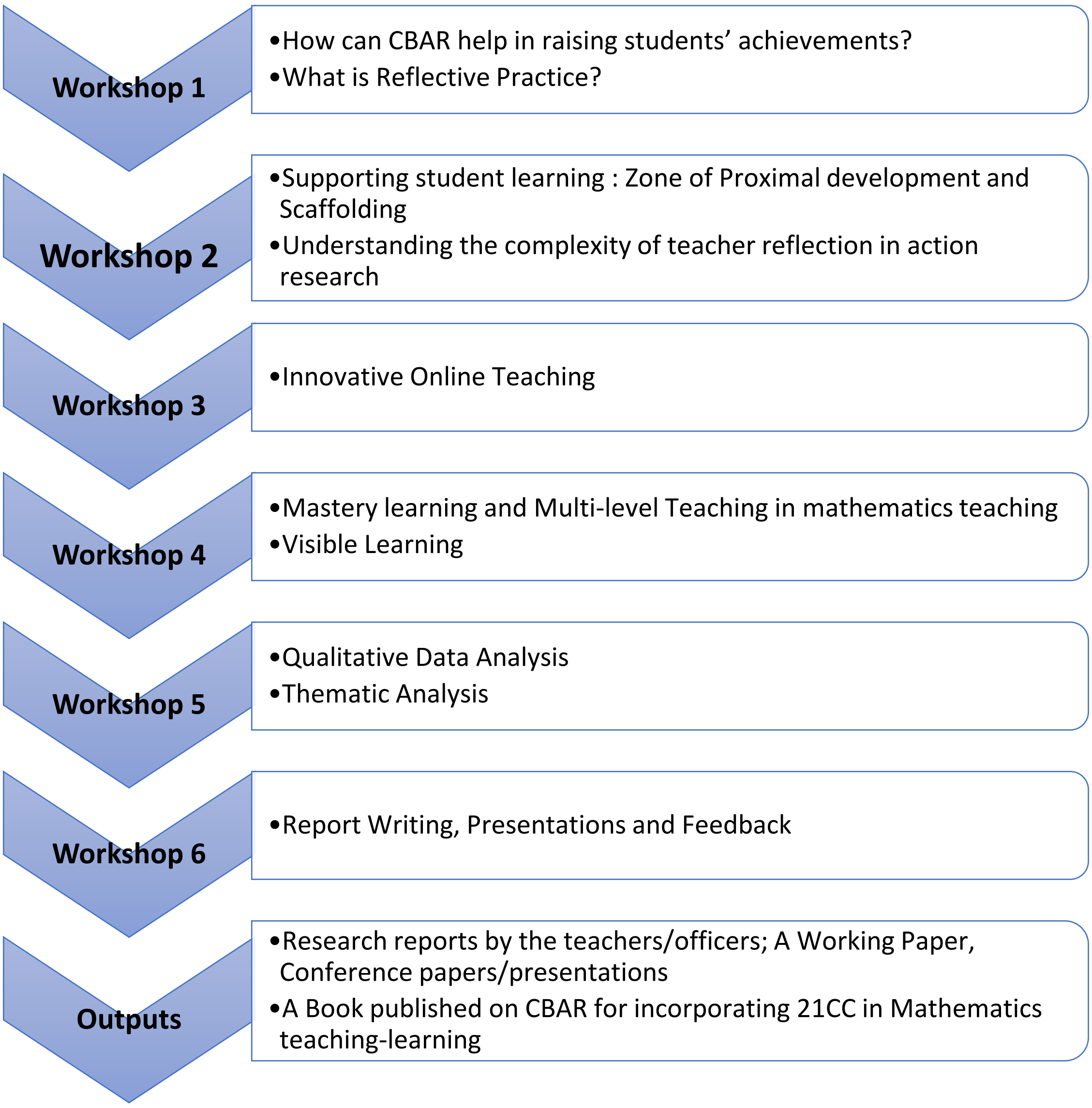


Figure 13: Six online workshops for providing additional support and feedback during the 2nd Cycle of CAR

By the time we reached this stage of the CAR project, I felt that our participants are focusing more on improving students' problem solving abilities and on incorporating 4Cs and metacognition. They appeared, frustrated about the students who are slow to acquire these abilities. I thought that we need to share some of our findings of phase 1 of the study with them to discuss the fact that in our mathematics classrooms, majority of students are struggling and achieving poorly. So, we need to think about ways and means of helping them in their learning. I thought we need to introduce the concept of Mastery learning by Benjamin Bloom and Vygotsky's ideas of zone of proximal development, assisted learning and scaffolding to sensitise our participants to design own strategies of helping such students based on theoretical understandings. Research team also agreed to this view and Walter suggested to introduce ‘visible learning’ in mathematics classes. We also felt that the participants have not grasped the meaning and practice of reflection properly.

Hence, we decided to discuss about ‘reflection’ in action research in detail in our online workshops. Prasad and Walter conducted a session on ‘Reflection’ in a subsequent workshop. We also felt that our participants and their students will be benefitted by a session on online teaching methods. Sakunthala suggested ‘Innovative online teaching’ and later conducted the workshop. At the end of the first online workshop, we asked the participants if they want any other topics to include in the workshops. Kavinda suggested ‘Qualitative data analysis’ and others also agreed to that and later utilised an online meeting to discuss that. In the sixth and final workshop we decided to focus on report writing. In these workshops we have used an interactive approach. Breakout rooms and screen sharing facilities in Zoom meetings and the limited number of participants in our sessions helped us to have good interaction with and among participants.

McNiff and Whitehead (2010) postulate that action research is a process of living one’s theory into practice. Over time, action researchers develop a deep understanding of how a variety of dynamic social and environmental factors interact with their practice to create complex patterns. In retrospection, I feel that CAR provided the research team and our collaborators, a space for mutual learning and to reflect on how to use participative ways in making decisions on teaching and curriculum and to enliven theories into practice. The approach is mutually beneficial for the university researchers and the collaborating teachers to improve their respective practices.

4.4 The Third Cycle of CAR:

How effective is the CAR and the CBAR implemented by teachers and officers and how can we share our understandings with important others?

Completion of the online workshop series at the end of February 2022, reminded us that it is time to start documenting and sharing the experiences of the CAR project as a whole and the CBAR by teachers. We decided to edit the working document 2 version 1, that we first developed in 2021 to describe the methodology, implementation process and the evaluation of our CAR and our overall reflections about the whole process. It is also time to start sharing our understandings with our stakeholders and the local and international research community. We have encouraged and guided our teacher collaborators to participate in local and international conferences and to write individual reports on their CBAR. Cycle 3 of CAR (see Figure 14) was dedicated for this purpose.

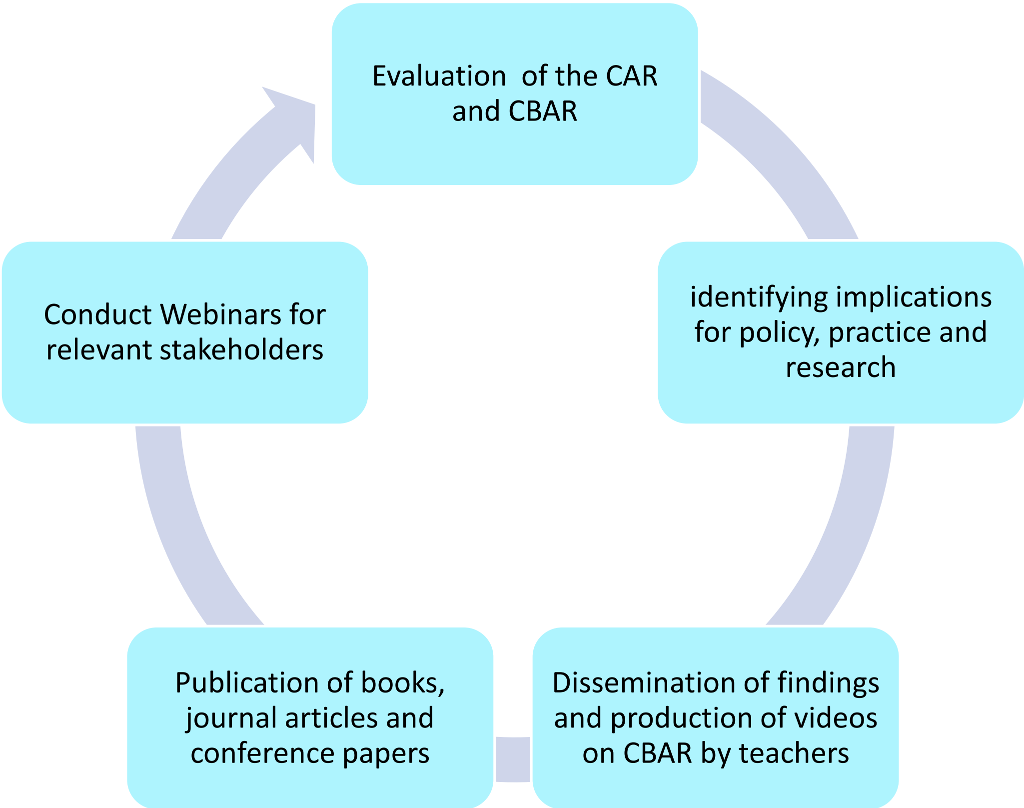


Figure 14: The 3rd Cycle of CAR: How effective is the CAR and the CBAR implemented by teachers and officers and how can we share our understandings with important others?

At the time of writing these final chapters, we have collected the reports written by our collaborating researchers. We have provided them feedback to revise the reports, if necessary, and guided them to write conference papers and journal articles. I also visited their schools with a team who produced a set of videos on their work to get a first-hand view of the work that they completed. A series of videos have been produced to disseminate key messages and a sample of the work done by the teachers and officers. At the end of the project we'll also conduct a few webinars to disseminate the findings of the CAR with higher authorities and mathematics teachers. A separate volume will be published on the CBAR conducted by our collaborators (the teachers and officers who participated in the CAR). Evaluation of the CAR, our reflections are reported in the next chapter.

4.5 Conclusion

In this chapter we have described how the CAR is implemented in three inquiry cycles in detail and reported our reflections in and reflections on our actions. In the next chapter, we evaluate the outcomes of our actions and report our final reflections to build a model for incorporating 21CC in Mathematics classrooms in Sri Lankan schools through CAR and discuss the implications of the model and our findings.

Chapter 5: Evaluation, Reflections and Implications

5.0 Introduction

In this Chapter, we report the university research team's evaluations and reflections about the CAR that we implemented over the past 2-3 years during COVID-19 Pandemic. Evaluation took place during the process of implementation as well as at the end of implementation. Formal and informal discussions with the collaborating teachers, officers and ISAs during workshops, progress review meetings, written reports by the collaborators, key messages written by the collaborators for video productions, and reflections recorded during video productions have been used to evaluate the success or otherwise of CAR. A model of CAR evolved during the process is depicted as a socially situated activity mediated by external pressures, inherent dilemmas in the activity system of the classrooms and knowledge, beliefs, assumptions and experiences of collaborating teachers and the university academics.

5.1 Evaluation and reflections

The key question for our evaluation of the success or otherwise of the CAR has been, *How effective were our collaborative actions in incorporating 21st Century Competences into mathematics teaching and learning in the targeted classrooms and what factors affected the process and outcomes of the CAR?*

We have used the evidence collected through our reflective journals, written reports of CBAR and key messages written by the collaborating teacher and officer researchers (See Annex 5), informal interviews with them and recordings of online workshops, progress review meetings and classroom videos. Following key themes emerged in the evaluation of the data.

1. Changes in students' learning, motivations, and social relationships
2. Changes in knowledge beliefs, assumptions, and practices of Teacher Collaborators
3. Changes in knowledge beliefs, assumptions, and practices of University Collaborators
4. Support for professional learning for collaborating teachers
5. Contextual demands and Dilemmas for collaborating teachers
6. External pressures
7. Support from school authorities

5.1.1 Changes in student learning, motivations, and social relationships in the classroom

The students' achievements were monitored by the pre-tests and post-tests conducted by the teachers during CBAR. All of them reported that most of their inventions were successful (See details in Key Messages-Annex 5) in improving students achievements to some extent. The interventions also helped to increase students' motivations and improve social relationships in the classrooms. One of the most successful interventions in improving student motivation and peer relations has been peer tutoring introduced by Sandamali. Following extracts from the interview I had with Sandamali explains her motive and strategy that she used.

> I noticed that, when I give additional exercises to them many students copy the answers of clever students. Rather than asking them to stop copying or punishing them I decided to instruct the students who used to copy from others to request the clever students to help them solve difficult problems. At the beginning those clever students were not ready to help others. I talked to them individually and assigned one clever student each to each of the student that need peer support for solving mathematical problems. Over time, the clever students became more enthusiastic about the process and conducted even Zoom lessons for their peers.
>
> - Sandamali (Interview, 26/01/23)

I had the opportunity to observe the process live in the classroom and when I asked peer tutees about the process at the end of the lesson, they expressed following views:

> "My tutor, Ruvini explains each step clearly and help me overcome difficulties in solving problems". - Asela

> “I’m so grateful to her (emotionally expressed), because of her and the madam, I like mathematics now”. Asela
>
> “ Ayodhya brings additional papers that her mother buys for her and share them with me. We solve problems together, where she helps me to attempt difficult ones. If both find it difficult, we ask from Madam. I am so grateful to Ayodhya for helping me to learn maths”.- Nimsara
>
> **“** I have scored below 25 earlier, and then Madam assigned Ruvini to help me, and my score increased to 40s, and in the last test I scored 60”. Sujith
>
> - (Extraced from Interviews with children, 26/01/23)

When I asked the peer tutors their views about the process, they expressed following views:

> “It helps me too to learn better. When I do a problem on my own, I learn it once only. But when I explain it to another, I must explain it in simple steps and sometimes do more of the similar problems. That helps me to learn a lot better.”

All students have been enthusiastic about the peer tutoring and learning process and a learning culture of the classroom appeared to improve a lot by Sandadmali’s actions. In other classrooms where the teacher researchers implemented CBAR, we noticed positive changes in student learning, motivations, and social relationships, since they created collaborative learning opportunities at some point of their research.

5.1.2 Changes in knowledge, beliefs, assumptions, and practices of Teachers:

The actions teachers implemented and their own evaluations about the success or the otherwise of those actions have encouraged them to persevere in the project. Individual reflections that they have reported in their individual reports, key messages, and teaching videos indicated that the whole process helped them change some of their initial beliefs about students, their ways of learning and own practices of teaching. For example, Darshani concluded in her report,

> This type of collaborative activities can be implemented in the classroom for motivating backward students for learning mathematics. I hope to implement similar activities in

> my classroom to improve students' scores and improving their understanding of mathematical principles. -Darshani's report on her CBAR, p23

Kavinda also concluded in his conference paper,

> As a teacher, I taught all students in the classroom as a one whole group without considering individual differences. However, by doing this study, I have realised that it is best done by recognising the level of children and planning activities that will awaken their thinking. I hope to use methods like this to teach in future classroom teaching. It is recommended to conduct further studies to practice probing prompting learning techniques so that students' attention is maximised, which can improve the development of students' metacognition toward solving mathematical problems.

Furthermore, collaborating teachers' research reports and key messages indicate how they have used the theoretical ideas and concepts conveyed to them through workshops in implementing their CBAR. In their key messages (See Annex II) and reports the teachers and officers indicated the underlying theoretical ideas and concepts on which their work has been based on. Among these were, metacognition, collaborative learning, inquiry based learning, levels of cognitive demand of mathematical tasks, peer tutoring and higher order thinking skills. They seem to have tried to enliven theories into practice.

5.1.3 Changes in knowledge beliefs, assumptions, and practices of University Collaborators:

I was the coordinator of this project and my role involved both administrative and research components. I had a permanent research assistant to assist me in administrative tasks and three colleagues to contribute to the research component. We have implemented this study while engaging in teaching and other normal administrative activities in the department of education and the faculty of Arts. My colleagues were also busy academics from the same department, and we found time amidst difficulties to implement the CAR project by working in extra hours in both weekdays and weekends.

The motivation for me to engage in this type of project came from my past engagements with action research (Wijesundera, 2002), continuing interest in education research, and improving the quality of general and higher education in Sri Lanka throughout my career in different capacities as a teacher, education officer, project officer at the National Institute of Education and a university academic. I believe equity and quality go together and therefore, to improve the quality of education in Sri Lanka it is necessary to ensure equity. Scrutiny of Mathematics, Science and Second language achievements of our students at the GCE(O/L) examinations in the past many years and the National assessments indicate the need for improvements in both quality and equity in education. To improve equity, we need to pay specific attention on providing equitable facilities to all children to thrive in learning mathematics, science and second language. Quality of education of an education system depends on the quality of teachers. I believe that the quality of teachers is reflected in their practice that depends on the complex interaction of their competence, beliefs, values, teaching learning situation and the support that they receive from relevant stakeholders. In this CAR project our theory of action had been, 'if we allow teachers to have a critical look at their classroom realities, identify issue(s) that they think they can address and take action in a systematic and a reflective manner to make the situation better, the teaching and learning in the mathematics classrooms will be improved'. We expected that their beliefs and practices will be changed in the process and that as university collaborators we would be able to provide theoretical and research guidance that they require in the process. At the same time, we expected that as university researchers we'll also engaged in reflective thinking and actions to improve our own professional practices.

The workshops, progress review meetings provided our collaborating teachers and officers spaces to present their actions and voice their concerns, experiences, beliefs, and values. University researchers also exchanged their views, beliefs and values with other team members and our collaborators. It was a novel experience for us to engage in this type of interactions with the practitioners about teaching and learning in mathematics classrooms. A key aspect among the many things that we learned in the process was that facilitation of CAR should be a democratic process where university researchers must be mindful about power relations and make an effort to develop more collegial relationships with the teacher researchers.

At the beginning, most of us tried to use a 'show and tell' type of guidance. Following type of exchanges took place in progress review meetings with our collaborators.

The university researcher said:

> 'I think you should do this…'

Then, later in the next session, the same researcher would ask the teacher collaborator,

> 'Did you do what I suggested last time? Or would say, 'You haven't done what I suggested you last time. Do you remember what I said?'

On the part of the collaborators, also, some of them asked the university researchers to suggest what they should do next. For example, I had the following encounter with a teacher collaborator.

Lochani: ‘I did this … gave them a set of word problems at level 2. All students achieved poorly in the exercise. Even the ones who achieved better in the class achieved poorly. I am trying to develop their ability to solve higher level problems. I am so sad. I am worried, why did I give them a such an assignment. What can I do now? Madam.’

At this point, I thought one thing that she can do is to select some poor achievers and privately ask them what went wrong? Why the problems were so difficult for them? To understand their problem clearly. I suggested that to her and she accepted my suggestion. However, in the next meeting I was stunned to hear Lochani saying,

> ‘Oh, Madam, I tried to talk to them privately, some of them said nothing and others said, ‘*Eka amaruyi Madam’* (That was too difficult) and nothing else.
>
> So, what shall I do now, Madam? I can’t figure out what I should do now.

My colleagues tried to ease the situation by cracking a joke and talking to her. In the meantime, I was thinking, what a blunder that I made here. I felt that I shouldn’t have suggested to her a single strategy. I should have suggested her many options for her to select. She seemed to have stopped thinking for herself, what she should do next. Did I violate her autonomy and confidence in making classroom decisions? I thought, I should admit my mistake openly.

Then I joined the discussion that my colleagues and Lochani were having, where she was saying I can use creative activities in the classroom. Then, I told her:

Subhashinie: I am sorry, I think I made a mistake here by suggesting what you should do. I shouldn’t have suggested a single strategy to you. You seemed to have stopped thinking yourself, what to do next and now you seem to be thinking that I have solutions to classroom issues. It’s not our purpose to make you dependent upon us. I think you came back to the right track when you say that you like to use creative strategies… Yes, please use your own creative activities. You may think about many ways of motivating the students to solve difficult word problems. Think about, ‘what can I do to encourage my students attempt to solve more difficult word problems?’

This encounter with the teacher helped me to rethink about the way that we guide the teachers in the CAR process. Showing and telling approach hinders teacher creativity and autonomy. Inquiring approach and a more collegial and collaborative approach is necessary to follow by the university researchers to promote professional learning by the teacher researchers.

University researchers reported in team meetings that they have gained new insights about teaching and learning through participating in the CAR process, and conducting workshops and consultancies on CAR methodology, providing guidance and feedback to participants. In addition to that, me and Walter who are responsible for teaching Mathematics teaching methods, and educational psychology courses at undergraduate and postgraduate diploma programmes have been able to develop specific insights on modifying existing curricula and teaching, learning and assessment strategies.

5.1.4 Support for professional learning

Teachers' professional learning was facilitated by workshops, progress review meetings and CBAR by teachers. Implementation of CAR inquiry cycles provided opportunities for the teachers to gain knowledge in 21st Century competencies, the concept, and processes of CAR and CBAR, develop skills in defining a researchable problem for their CBAR, data collection analysis, reflection and evaluation of their own actions as well and to change their attitudes, values and practices by reflecting on their actions and consequences. For example, Sandamali's account of her CBAR and her key message indicate, how she attempted to motivate students who were not interested in learning mathematics in her classroom and use peer tutoring (using more knowledgeable peers to support learners who are lagging). Moreover, her actions seem to have helped the students in her Grade 7 class to change their behaviours and values. Following extract from the telephone interview I had with Sandamali illustrates this latter point.

Subhashinie: So, in your CBAR what were the key things that you attempted to change?

Sandamali: My students considered mathematics as a very difficult subject. They did not do the exercises in the textbook. They easily forgot what is taught. So, I thought I should start with implementing simple activities that will motivate them for learning maths. After implementing those activities, I gave a post test. I have grouped the students based on the post-test results. Then, I encouraged the students in the lower bands to try to achieve at least 5 more marks in the next test. I found that those students have difficulties in using mathematical operations and understanding word problems. I thought I would use collaborative learning activities to help them. I wanted to use clever students to support their peers who have difficulties in learning maths.

Subhashinie: How did you motivate the clever children to support their peers?

Sandamali: At the beginning such children were reluctant to help others. They were selfish. They will not show their work to others. But then I explained them that they are clever now because they have helped others and did good *kamma* in their past lives. So, if you help them now, you will become cleverer. They are young children and they complied with my request and now, they are doing better than I expected. I assigned one low performing student each to each of the clever student. The clever ones became motivated to help their peers by giving additional exercises, supporting them solve problems in the textbook, and even conducting zoom classes. Sometimes they complain to me if their assigned peer is absent. They are so eager to help now. One child gives lattice exercises to other children when they have free times in the classroom.

Telephone Interview- 20/01/2023

Sandamali's actions reported above seem to have changed her beliefs about her students and her practices. Sandamali further explained,

> " I have been teaching mathematics in Grades 10-11 classes. For this project I have undertaken a Grade 7 class last year. Most children in my Grade 10-11 classes are so poor in maths, and I conduct extra classes and do a lot of past papers with them. But still, they don't do well in the GCE (O/L) exam. Rote learning doesn't help them. They don't have a good understanding of basic math concepts. I was frustrated. However, after working with these junior students, I feel that these Grade 8 children will be doing much better, since I gave them opportunities to learn maths with understanding. I want to continue to undertake at least one class from the junior secondary grades.

Interview 26/01/2023

Sharing the collaborating teacher's experiences with us and other collaborators (other teacher researchers, officer and ISA in the workshops and progress review meetings) have provided them opportunities for mutual learning and academic, social, professional and emotional support to persevere in their CBAR.

5.1.5 External pressures:

The teachers designed and implemented their CBAR projects in a challenging environment during COVID 19 Pandemic, where they had difficulties in interacting with students due to lockdowns, lack of digital infra structure and or students' lack of devices.

> "I have selected 7 schools and found that 50% of students can access Zoom lessons. 30% WhatsApp and 20% do not participate in any of these."
>
> Polgaspitiya (Progress review meeting 1)

Some of them had opportunities to use synchronous modes of online teaching where they had some opportunities for live interactions with the students. Others used asynchronous modes of teaching and learning via social media platforms since there were connectivity problems and lack of computers and/or other devices for the students. Few others had to rely on physical interactions which were limited due to COVID-19 related restrictions, school closures and the lack of digital infra-structure.

School closures due to pandemic situation, teachers' trade union action in 2021 and political and economic crisis in 2022 also affected the CAR adversely.

> We are asked to do two lessons in one day to catch up the lost time and work. I have 34 students in my class. It is difficult to focus on students who are not doing well in maths.
>
> (Lochani- Progress review meeting 1)

Despite these issues all eight participants participated in online workshops and progress review meetings with enthusiasm. They appeared interested in seeing improvements in students' engagement and achievements even if there were minor improvements.

5.1.6 Contextual Demands and dilemmas faced by the teachers.

Time constraints and emphasis on increasing test scores affected the teacher actions and their professional learning adversely. Teacher responses in the workshops and progress review meetings reveal the dilemmas faced by them.

> In my school about 50% of students score below 40 in term tests. Teachers know the problem and reasons for that. I think it is the exam system, only the knowledge is tested. Skill is not tested. If the teacher has some autonomy the situation will change a lot.
>
> (Kavinda- Progress review meeting 2)

Others also expressed similar views. They said student active teaching methods are necessary to promote students' understanding of mathematical concepts and acquiring 21st century competencies, but they must cover the syllabuses fast and prepare students for examinations by giving more practice activities that promote rote learning. Here, the teachers face the dilemma of *Promoting meaningful active learning vs encouraging passive rote learning*. Teacher actions and the inquiry process that they adopt in the classroom depends on the way the teachers resolve this kind of contextual dilemmas as indicated in the following extract from Kavinda's reflections recorded during the video production.

> I find this type of student active methods of teaching are very useful in improving student motivation. I suggest other teachers also to use this type of methods at least once a week. Children like this type of activities. So, even if these types of activities take more time and effort by the teacher, it is worth to use them at least once a week since they help students learn better.

5.1.7 Support from school authorities

Principals supported the teachers CBAR by allowing them to participate in the CAR and providing necessary facilities in schools for teacher actions, and project activities such as video recording of lessons. When I meet the principals in our visits during video production, all principals appreciated innovative work done by the teacher researchers in their schools.

5.2 Final Reflections: a CAR model for incorporating 21CC in the mathematics classroom

In this CAR our intention was to make our teachers 'inquiring teachers' (Schnellert and Butler, 2014) who will engage in iterative cycles of action and reflection. For Schnellert and Butler, Mindful inquiry starts with defining a problem and then framing it as a more specific, personally relevant question. Then, teachers draw on resources to advance their professional learning, plan how they might take up ideas and enact them in practice, monitor progress towards goals, and adjust as needed (Schnellert and Butler, 2014, p.42). The way I thought about in the above encounter with the teacher researcher, seem to match with the Schnellert and Butler's view of inquiring teachers. In this type of situations as collaborating researchers we need to support the teachers to redefine their problem as a more specific, personally relevant question rather than suggesting solution(s). Following Schnellert and Butler (2014)'s situated model of inquiry, and based on our own experiences, reflections and evaluations on the CAR lead us to conceptualise our CAR process as per the model set out in Figure 15.

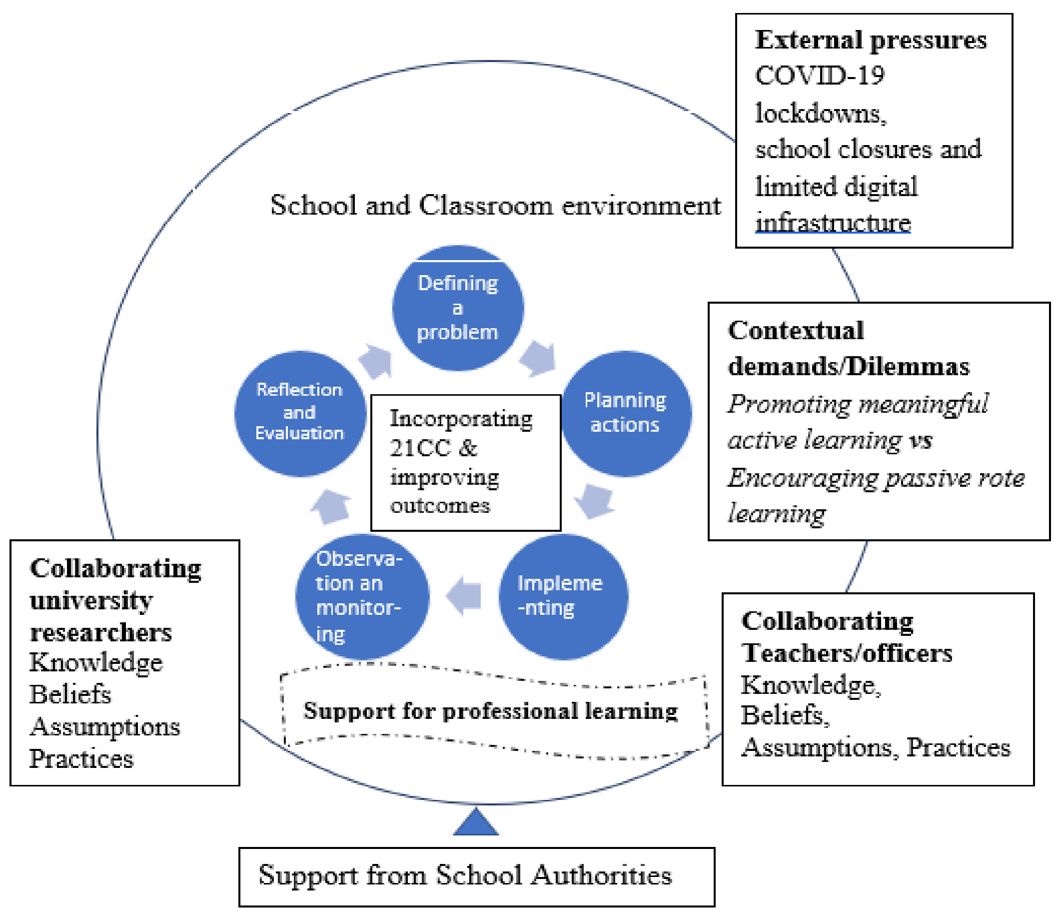


Figure 15 : A CAR model for incorporating 21CC in mathematics classrooms

As depicted in the model the inquiry process adopted by the teachers is the key process to incorporating 21st century competencies among mathematics learners in the classroom which is situated in specific social context of the school and the classroom. Mathematics Teacher's teaching and professional learning as well as students' learning are affected by the inquiry process that he/she engages, and other mediating factors depicted in the model. Support from the school authorities is pivotal for implementing the inquiry process by the teachers. Teacher's professional learning is facilitated by their own autonomous actions and reflections as well as the supportive interactions with the collaborating researchers from the university. In other words, teacher autonomy and appropriate professional support are the necessary conditions for the success of CAR. Sustainability of the inquiry process by the teachers depends on how they resolve the emerging contextual dilemmas and the support that they receive from the school authorities. External pressures indicated in the model also affected the teachers' inquiry process. We believe that the model evolved through the CAR process can be used by the school and zonal authorities to reforming education at classroom level and incorporating 21 CC in mathematics classrooms .

5.3 Implications for policy, practice, and research

Current study confirms that a CAR approach is useful in incorporating 21CC among mathematics learners at the junior secondary level. The findings help us to argue that teacher professional learning and improving students outcomes must be the focus of reforming education at the classroom level. As depicted in the model for incorporating 21CC in the mathematics classroom, teachers need support from school authorities and from collaborating researchers to implement a systematic inquiry approach for improving student learning in their classrooms. Therefore, provincial, zonal, and school authorities need to consider adopting collaborative approaches in reforming education. Schools with the support of local experts can initiate CAR studies as part of their School Based Teacher Professional Development (SBTPD) programmes. Zonal and provincial authorities can support schools by providing necessary funding and mobilising necessary expert services. The experts must work with teachers in a collegial and respectful manner to facilitate mutual learning and for developing successful collaborations. The insights developed through the current research are also useful in reforming teacher education where more opportunities must be provided for the trainees to engage in inquiry based learning and collaborative learning where they can examine their beliefs, values and practices. Finally, we suggest that similar approaches of CAR may be useful in incorporating learning and innovation skills into other subjects in the school curriculum.

5.4 Conclusion

In this CAR project our theory of change had been, 'if we allow teachers to have a critical look at their classroom realities, identify issue(s) that they think they can address and take action in a systematic and a reflective manner to make the situation better, the teaching and learning in the mathematics classrooms will be improved'. We expected that their beliefs and practices will be changed in the process and that as university collaborators we would be able to provide theoretical and research guidance that they require in the process. In the evaluation of our work, it is emerged that CAR approach have been useful in incorporating 21st century competencies into teaching and learning in mathematics classrooms. Our understandings generated through the study helped us to develop a socially situated model for incorporating 21CC in mathematics classrooms and for improving professional learning of mathematics teachers. Teacher autonomy and appropriate professional support are the necessary conditions for the success of CAR in reforming education at classroom level. We further suggest that the CAR model that we developed is useful for implementing similar programmes at provincial, zonal and school levels to improve teacher professional learning as well as student learning.

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Annex 1: The Agenda of the 4-day Workshop

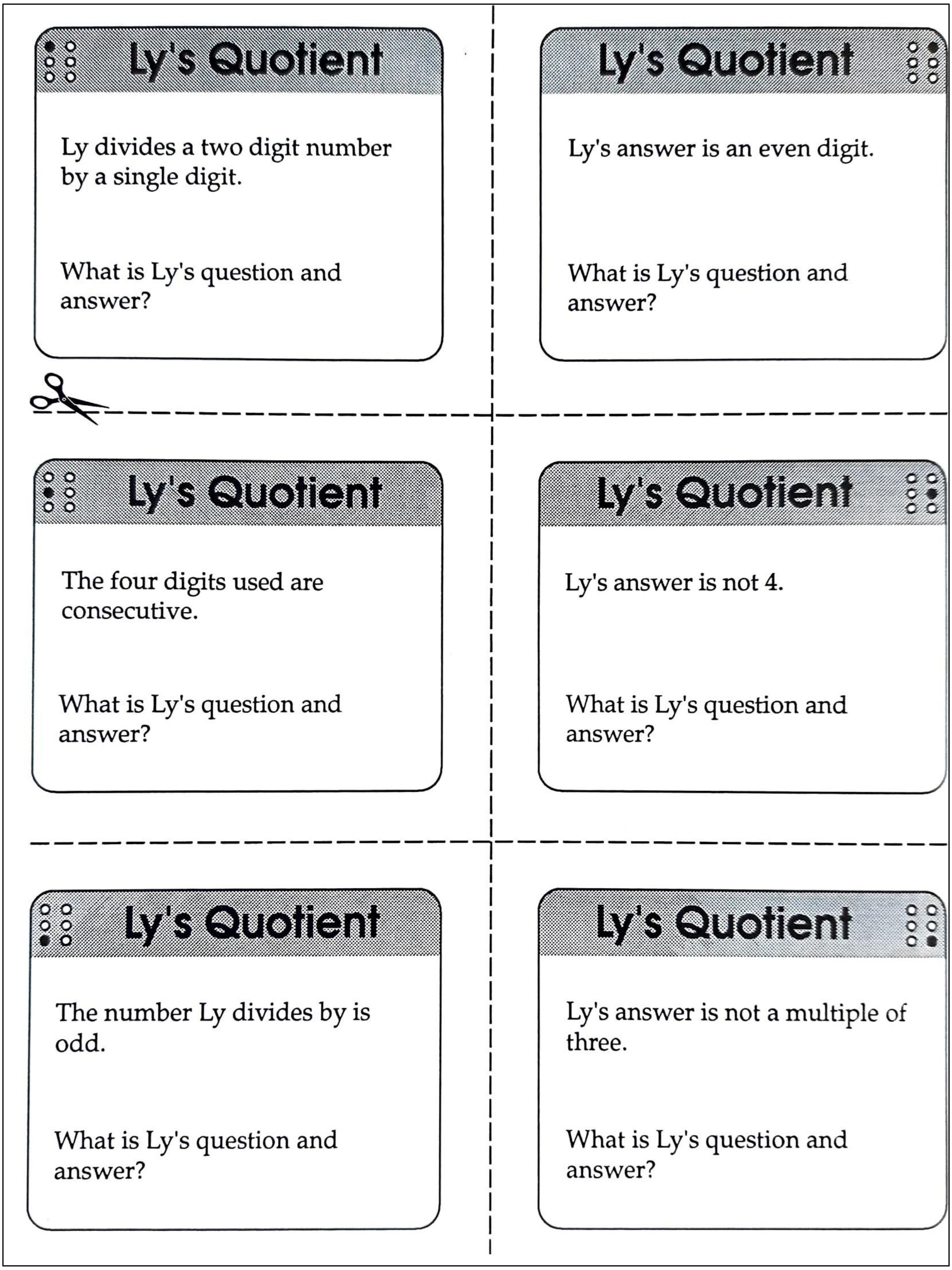
Workshop for the Resource Persons on the 21st Century Skills in the Mathematics classrooms-From 02/10/2019 to 05/10/2019

Date	Time	Theme
Day 01- **Wednesday 02/10/2019**	09.00 a.m. – 09.45 a.m.	Inauguration
	09.45 a.m. – 10.00 a.m.	Tea
	10.00 a.m. – 01.00 p.m.	**Mathematics Education- 21st century competencies** *Session 01:* o What are 21st century competencies? o How are they developed?
	01.00 p.m. – 01.45 p.m.	Lunch
	01.45 p.m. – 03.45 p.m.	**Mathematics Education- 21st century competencies** *Session 02:* o Identification of 21st century competencies and skills for infusion in their mathematics teaching and learning at the junior secondary level. o Bedrock of Mathematics lessons - mathematical tasks and classroom discourse
	03.45 p.m.	Tea
Day 02- Thursday 03/10/2019	09.00 a.m. – 12.00 p.m.	**Thinking skills and heuristics (Problem solving and problem posing)** *Session 01:* o Teaching for understanding o Collaborative problem solving
	10.30a.m.	Working Tea
	12.00 p.m. – 01.00 p.m.	Lunch
	01.00 p.m. – 03.00 p.m.	**Thinking skills and heuristics (Problem solving and problem posing)** *Session 02:* o Collaborative problem solving – cont. o Characteristics of tasks suited for problem solving and posing / examination of tasks from textbooks
	03.00 p.m.	Tea

Day 03- **Friday** **04/10/2019**	09.00 a.m. – 12.00 p.m.	**Reasoning and communication** *Session 01:* o Characteristics of tasks that engage students in reasoning and communication – the "What" strategies o What number makes sense? o What's wrong?
	10.30 a.m.	Working Tea
	12.00 p.m. – 01.00 p.m.	Lunch
	01.00 p.m. – 03.00 p.m.	**Reasoning and communication** *Session 02:* o What would you do? o What if? o What's the question if you know the answer?
	03.00 p.m.	Tea
Day 04- **Saturday** **05/10/2019**	09.00 a.m. – 12.00 p.m.	**Reasoning communication and empowering the learner** *Session 01:* o Metacognition and strategies for engaging students in reflection o teacher and student actions for nurturing self-regulated learning
	10.30a.m.	Working Tea
	12.00 p.m. – 01.00 p.m.	Lunch
	01.00 p.m. – 03.00 p.m.	**Reasoning communication and empowering the learner** *Session 02*: o Classroom talk – monologic versus dialogic o Facilitation of class discussions
	03.00 p.m. – 03.30 p.m.	Closure and Tea

Annex 2: Samples (2-8) of Classroom Activities

Sample 2 (Gould, 1993, p. 61)



Ly's Quotient

Ly divides a two digit number by a single digit.

What is Ly's question and answer?

Ly's Quotient

Ly's answer is an even digit.

What is Ly's question and answer?

Ly's Quotient

The four digits used are consecutive.

What is Ly's question and answer?

Ly's Quotient

Ly's answer is not 4.

What is Ly's question and answer?

Ly's Quotient

The number Ly divides by is odd.

What is Ly's question and answer?

Ly's Quotient

Ly's answer is not a multiple of three.

What is Ly's question and answer?

Sample 3 (Gould, 1993, p. 62)

Linh's Data

All the scores are positive integers and there is an odd number of scores.

Help your group find Linh's data.

Linh's Data

The total of all the scores is 22.

Help your group find Linh's data.

Linh's Data

The median score is 5.

Help your group find Linh's data.

Linh's Data

The mode of Linh's scores is 7.

Help your group find Linh's data.

Linh's Data

The range of the scores is 6.

Help your group find Linh's data.

Linh's Data

The biggest score in Linh's data is 7.

Help your group find Linh's data.

Sample 4 (Gould, 1993, p. 63)

Rau's Data

The mode of Rau's data is 1.

Help your group find Rau's data.

Rau's Data

There are four scores in Rau's data and they are all integers but they are not all positive.

Help your group find Rau's data.

Rau's Data

The median of Rau's data is 0.

Help your group find Rau's data.

Rau's Data

The mean (average) of the scores is -1.

Help your group find Rau's data.

Rau's Data

The range of the scores is 6.

Help your group find Rau's data.

Rau's Data

Half of the scores in Rau's data are negative.

Help your group find Rau's data.

Sample 5 (Gould, 1993, p. 72)

Ryan's Sum

Ryan's sum is formed by adding a two digit number and a single digit.

What is Ryan's sum?

Ryan's Sum

Only two different **digits** are used in the question and in the answer.

What is Ryan's sum?

Ryan's Sum

The digit in the units place in the answer is the same as the digit in the tens place in the question.

What is Ryan's sum?

Ryan's Sum

The digit in the tens place in the answer is the same as the digit in the units place in each number in the question.

What is Ryan's sum?

Ryan's Sum

The answer to Ryan's sum is an even number.

What is Ryan's sum?

Ryan's Sum

The answer written backwards is the two digit number in the question.

What is Ryan's sum?

Sample 6 (Gould, 1993, p. 74).

Carole's Product

Carole multiplies a two digit number by a one digit number.

Find Carole's answer!

Carole's Product

The numbers Carole multiplies contain only the digits 3,5 and 7 each used once.

Find Carole's answer!

Carole's Product

Carole's answer contains only odd digits.

Find Carole's answer!

Carole's Product

Carole's answer has three digits and none are repeated.

Find Carole's answer!

Carole's Product

Carole's answer is greater than 200.

Find Carole's answer!

Carole's Product

The answer is not a multiple of five.

Find Carole's answer!

Sample 7 (Gould, 1993, p. 73)

What's the Difference

A three digit number is subtracted from a three digit number to give a three digit answer.

Find the question and the answer.

What's the Difference

All the digits from 1 to 9 are used exactly once in forming the question and answer.

Find the question and the answer.

What's the Difference

All the digits in the answer are even.

Find the question and the answer.

What's the Difference

Both the numbers in the question are odd.

Find the question and the answer.

What's the Difference

The answer is less than 500.

Find the question and the answer.

What's the Difference

If you multiply the first two digits in the answer you get the last digit.

Find the question and the answer.

Annex 3: Modification of textbook tasks (Additional examples)

Strategy 1: What number makes sense (Kaur & Yeap, 2009, pp.14-15)

Example:

EXAMPLE 2

A typical textbook question:

I have a rectangular study table.
The length is 80 cm and the breath is 60 cm.
I also have a rectangular exercise book.
The length is 16 cm and the breath is 15 cm.
How many exercise books do I need to cover the top of my table?

What number makes sense?
Read the problem. Look at the numbers in the box.
Put the numbers in the blanks where you think they fit best.
Read the problem again, do the numbers make sense?

My study table

My study table has a rectangular table-top.

It is ______ cm long and ______ cm wide.

The area of the table-top is ______ cm^2.

My exercise book is rectangular in shape too.

It is ______ cm long and ______ cm wide.

To completely cover the top of my table with exercise books,

I need ______ exercise books.

15	16	20	60	80	4800

Strategy 2: What's wrong? (Kaur & Yeap, 2009, pp. 22-23)

Example

EXAMPLE 2

Mr Tan's trip

A taxi charges:

For the first 1.5 km	$2.40
For every additional 100 m	$0.10

Mr Tan paid $12.00 for his taxi ride.
He thought his trip was 14.4 km long.

David's solution:- $12.00 – $2.40 = $9.60
$9.60 ÷ $0.10 = 9.6
9.6 × 1.5 = 14.4

There is something wrong with David's solution.

1. Show how you would find the answer to the problem.

2. Explain the mistake in David's solution.

Strategy 3: What would you do? (Kaur & Yeap, 2009, p. 30)

Example

EXAMPLE 2

Birthday candles

Raju has 8 large and 5 small candles.
He has to put candles on a birthday cake to celebrate his grandfather's 64th birthday.

1. How many candles of each type could Raju put on the birthday cake?

2. Explain your reasoning.

Strategy 4: What questions can you answer? (Kaur & Yeap, 2009, p 37-38).

Example

EXAMPLE 2

The Exhibition

An average of 215 people visited a 4-day exhibition on the first three days. Another 310 people visited the exhibition on the fourth day.

Write two questions you can answer about the visitors to the exhibition.

1. Question 1

   ____

2. Question 2

   ____

3. Find the answer to your questions.
   Show your work.

   ____

Strategy 5: What's missing? (Kaur & Yeap, 2009, pp. 45-46)

Example 1

EXAMPLE 1

Donuts

Mary bought 7 boxes of donuts for her class party.
She paid $35 for the 7 boxes.
How much did each donut cost?

Can you find the cost of a donut?

Use the following prompts to guide you.

(a) What information do you know from the problem?
(b) What else do you need to know to solve the problem?
(c) Pick a number that shows how many donuts might have been in a box.
How much would each donut cost?

How much would each donut cost?

Show your workings.

Example 2

EXAMPLE 2

Rectangular picture

The area of a rectangular picture is 108 cm^2.
Find the length and perimeter of the picture.

Can you find the length and perimeter of the picture?

Use the followings prompts to guide you

(a) What information do you know from the problem?
(b) What else do you need to know to solve the problem?
(c) Pick a number that represents the breath of the rectangular picture.

How long is the picture?

What is the perimeter of the picture?

Strategy 6: What if? (Kaur & Yeap, 2009, p. 53)

Example 1

EXAMPLE 1

Cookies and Boxes

Mrs Tan baked 24 cookies.
Each box holds 4 cookies.
At least how many boxes are needed to hold all the cookies?

What if Mrs Tan baked 30 cookies?
What if each box can hold 5 cookies?
What if each box can hold up to 4 cookies?

Generate another 3 "What if" tasks and answer them.
Look out for any interesting observation/pattern.

Example 2

EXAMPLE 2

Beans and bag

A sack of green beans had mass of 7kg.
Chee Keong divided the beans equally into 3 bags for his customers.
What was the mass of each bag?

What if the mass of green beans is 10 kg?
What if there are 5 bags?
What if Chee Keong divided the beans in the ratio 1 : 2 : 3?

Generate another 3 "What if" tasks and answer them.
Look out for any interesting observation/pattern.

Strategy 7: What's the question if you know the answer? (Kaur & Yeap, 2009, pp. 60-61)

Example 1

EXAMPLE 1

Red & white chalk

Mr Lee had 3 boxes of red chalk and 8 boxes of white chalk.
Each box contained 5 pieces of chalk.

1. What's the question if the answer is 40 ?

   ____

2. What's the question if the answer is 15 ?

   ____

3. What's the question if the answer is 11 ?

   ____

4. What's the question if the answer is 3 : 8 ?

   ____

Example 2

EXAMPLE 2

Eggs sold by Mr Ali

The table shows the number of eggs sold by Mr Ali on each day.

Day	Monday	Tuesday	Wednesday	Thursday
Number of eggs sold	135	150	110	185

1. What's the question if the answer is Wednesday ?

____

2. What's the question if the answer is 580 ?

____

3. What's the question if the answer is 75 ?

____

4. What's the question if the answer is 145 ?

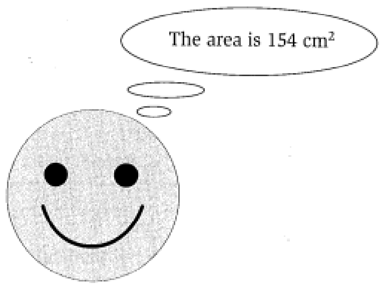
____

Strategy 8: What is the question? (Kaur & Yeap, 2009, pp. 68-69)

EXAMPLE 1

Topic: Area of plane figures

One - Five - Four



1. What could the question be?

____

2. My solution is :

____

Example 2

EXAMPLE 2

Topic : Speed

Average speed

This is all that is left of your homework.
Your dog ate the question and also parts of the solution.

Average speed = 65 km/h

What could the question be?
Your teacher wants you to write a question.
You must also provide a complete solution.

1. The question is:

____

2. My solution is:

____

Annex 4: Reflections of participants at the end of the workshop

Trainer	What did I learn in this workshop?	How useful is that learning to improve my practice?	How can I use them to teach other teachers?
A	• I was able to update my knowledge. • This gave me new ideas to implement in the classroom. • I understood the need to be a knowledgeable and skilled teacher who works with dedication.	• I learned how to plan lessons to develop students' skill and creativity. • How to select questions according to the topic and design activities • How to implement four levels of mathematical tasks	In consultation with Deputy Director (Maths) • To inform – Principals Subject Coordinators Maths Teachers. • To organise workshops 6 – 11 secondary classes 1 – 5 primary grades • To monitor progress • To design lessons to promote students' knowledge and skills through activities
B	• Importance of maths for the students • Teaching techniques • Different countries using their teaching methods. • How to solve the student's real-world problem • Making the answer → questions	• Solving methods • Levels of the questions for students' level	• Teaching technique • Different teaching methods

C	• How to achieve the goal • How to deal the students in 21st century • How to behave the 21st century students • New updates, research, and innovations. • Are we prepared to teach 21st century students	• Some short methods very useful to us • Group activities and discussion methods very useful to us	• 4I Teacher Development we can apply to the teachers. • How to arrange the presentation • SPUR methods • Teaching for understanding • Knowledge Vs. understanding
D	• Identified competencies and skills in Singapore's education. • 21 වැනි ශතවර්ෂයේ junior secondary සිසුන්ගේ ගණිත නිපුණතා හා හැකියා වර්ධනය වන ආකාරයට ඉගෙනුම් ඉගැන්වීම් සංවිධානය කළ යුත්තේ කෙසේද? (How to organise teaching and learning at junior secondary level to develop 21st century mathematics competencies among students)	• To motivate student in classroom • To make different level problems for students • To prepare different creative activities using the textbooks and syllabus	• I will introduce them the above factors. • I hope to make different creative activities with them. • I help them to motivate students
E	• සහයෝගයෙන් යුතුව ඉගෙනීම (Collaborative	• Level 3 සඳහා ක්‍රියාකාරකම් (activities	• SBTD වැඩසටහන් මඟින් ගුරුවරුන්ට ලබා දීම (Use

	learning) • ක්‍රියාකාරකම් පාදක ව ඉගෙනීම (Activity based learning) • මෘදු කුසලතා සංවර්ධනය (Soft skills) • ආචාර ධර්ම(Ethics) • ගණිත පර්යේෂණ සිදු කිරීම (Mathematics research) • තාක්ෂණය භාවිතය මගින් ඉගෙනීම (Use of technology in learning • ගවේෂණ කාර්යය (Exploratory tasks)	for level 3) • පංතිය තුළ ක්‍රියාකාරකම් සිදු කරන විට මනෝවිද්‍යාත්මක න්‍යායාත්මක කරුණු අනුගමනය කිරීම (ප්‍රායෝගික බුද්ධිය) (use of psychological and theoretical facts in implementing classroom activities) • පංතිය ක්‍රියාකාරකම් සිදු කරන විට සුහග සිසුන් සඳහා ගැටළු විසඳීමේ ක්‍රමයේ දී විවිධ ආකාරයන් පිළිතුරු ගත හැකි ප්‍රශ්න ලබා දීම සඳහා ප්‍රශ්න සැකසීමට (Preparing different kinds of problem solving activities for gifted students) • සිසුන් තුළ ප්‍රායෝගික කුසලතාවයන් වර්ධනය වන ආකාරයට පාඩම් සැලසුම් සැකසීම (Preparing lesson plans to develop practical skills among students) • සිසුන් තුළ අධිප්‍රජානන කුසලතා ඇති කිරීමට (Developing metacognitive skills among students)	for SBTD) • ගණිත විෂයයේ සමහර පාඩම් ඉගැන්වීම පිළිබඳ අනෙකුත් ගණිත ගුරුවරුන් සමඟ සාකච්ඡා කිරීම(To discuss with other maths teachers) • සමෝධානිත විෂයමාලාව පාසල තුළ ක්‍රියාත්මක කරන අවස්ථාවලදී • ගණිත දින, ගණිත සැසි, ගණිත කඳවුරු පවත්වන අවස්ථාවලදී (Useful for ‘mathmatics days’ , camps and seminars)

Annex 5: A Sample of Key messages of CBAR

Key Message 1- Mathematical problem solving using metacognition

by Kavinda Wijethunga

<u>ගණිත ගැටළු විසඳීමට අධිප්‍රජානනය භාවිතා කිරීම</u>

රජයේ පාසලක ගණිත ගුරුවරයකු වශයෙන් වසර දොළහක කාලයක් තිස්සේ සේවය කරනවා. එම කාල සීමාව තුළ 6 ශ්‍රේණියේ සිට 13 ශ්‍රේණිය දක්වා සිංහල සහ ඉංග්‍රීසි යන මාධ්‍යය දෙකෙන්ම ගණිතය සහ සංයුක්ත ගණිතය යන විෂයයන් ඉගැන්වීමේ කටයුතු සිදු කර තිබෙනවා.

ගුරුවරයකු වශයෙන් මට හැකි උපරිම අයුරින් මෙම කාල සීමාව තුළ සිසුන්ට ගණිත ඉගැන්වීමේ කටයුතු සිදු කරනවා.

එහෙම උනත් වාර විභාගවලදී සියලුම සිසුන්ට මම බලාපොරොත්තු වෙන විදිහට ඉහළ ලකුණු ලබා ගන්න බැරි වෙලා තියෙනවා.

සාමාන්‍ය පෙළ වගේ ජාතික මට්ටමේ විභාගවලදීත් අනිත් විෂයයන් සමඟ සංසන්දනාත්මක ව බැලුවම අඩු ලකුණු තමයි ගණිතයට අරගෙන තියෙන්නෙ.

මේ තත්ත්වය මග හැර ගන්න පාසලෙන් ක්‍රියාත්මක කරන ලද අමතර පන්ති, ප්‍රශ්න පත්‍ර සාකච්ඡා කිරීම් වැනි උපක්‍රම භාවිතා කරත් අපේක්ෂිත විදිහට ළමයින්ගේ ප්‍රතිඵල වර්ධනය කර ගන්න බැරි වෙලා තියෙනවා.

ළමුන්ගේ ගණිත සාධනය වැඩි කර ගැනීමට විවිධ උපක්‍රම අත්හදා බැලීම සිදු කරමින් සිටින අතරතුර පේරාදෙණිය විශ්වවිද්‍යාලයේ අධ්‍යාපන අධ්‍යයන අංශය මඟින් AHEAD *ව්‍යාපෘතිය යටතේ සිදු කරමින් යන පර්යේෂණයකට සම්බන්ධවීමට අවස්ථාව මට ලැබෙනවා.*

එම ව්‍යාපෘතිය යටතේ පවත්වනු ලැබූ සිව්දින වැඩමුළුවකට මම සහභාගී වුණා. එහිදී විසි එක්වැනි ශත වර්ෂයේ කුසලතා පිළිබඳව දැන ගැනීමට ලැබෙනවා. ඒ අතරතුර අධිප්‍රජානනය (Metacognition) *යන සංකල්පය ගණිත ගැටළු විසඳීම සඳහා භාවිතා කළ හැක්කේ කෙසේද යන අවබෝධය මා හට ලබා ගැනීමට හැකියාව ලැබෙනවා.*

*"කෙනෙකුට තම ප්‍රජානන ක්‍රියාවලි, ප්‍රජානන ක්‍රියාවලිවල ඵල හෝ ප්‍රජානන ක්‍රියාවලිවලට සම්බන්ධ ඕනෑ ම දෙයක් පිළිබඳ ව පවතින දැනුම අධිප්‍රජානනයයි" (*Flavell, 1976).

එම වැඩමුළුවේ දී ලබා ගත් දැනුම භාවිතයෙන් ගණිත ගැටළු විසඳීම සඳහා අධිප්‍රජානනය භාවිත කළ හැක්කේ කෙසේ දැයි විමසා බැලීමට කාර්යමූල පර්යේෂණයක් සැළසුම් කළා.

මගේ කාර්යමූල පර්යේෂණයේ අරමුණ උනේ '8 ශ්‍රේණියේ සිසුන්ගේ අධිප්‍රජානන හැකියාවන් වර්ධනය කිරීම මඟින් ගණිත ගැටළු විසඳීමේ හැකියාව වැඩිදියුණු කිරීම සඳහා අධිප්‍රජානන සහය ලබා දීමේ සඵලතාවය විමසා බැලීම යි'.

චක්‍ර හතරකින් සමන්විත වූ මෙම ක්‍රියාමූලික පර්යේෂණයේ දී, පළමු චක්‍රයේ දී සිසුන්ගේ පවතින කාර්ය සාධනය සහ අධිප්‍රජානන දැනුම පිළිබඳ අවබෝධයක් ලබා ගත්තා.

දෙවන සහ තෙවන චක්‍රවල දී සිදුකරන ලද ක්‍රියාකාරකම් රාශියකින් සිසුන්ගේ කාර්ය සාධනය වැඩිදියුණු කිරීම සඳහා විවිධ අධිප්‍රජානන උපාය මාර්ග භාවිතා කලා.

මෙම චක්‍ර දෙකේදී භාවිතා කළ ක්‍රියාකාරකම් වලංගුභාවය විද්වත් මණ්ඩලයකට ඉදිරිපත් කර අනුමත කර ගැනීමෙන් අනතුරුව භාවිතා කරන ලදී.

හතරවන චක්‍රයේ දී අධිප්‍රජානන දැනුම වැඩි වී ඇත් ද, කාර්ය සාධන වැඩි වී ඇත් ද යන්න පරීක්ෂා කලා.

එමඟින් අධිප්‍රජානන උපාය මාර්ග භාවිතයෙන් සිසුන්ගේ ගණිත ගැටළු විසඳීම යම්තාක් දුරට වර්ධනය කළ හැකි විය.

අවබෝධයකින් තොරව සංඛ්‍යාත්මක පිළිතුරු ලබා ගන්නවා වෙනුවට දීර්ඝකාලීන ඉගෙනුමක් සඳහා අධිප්‍රජානනය උපකාරී වන බව, සිසුන් සමඟ සාක්ච්ඡා කරන විට පෙනෙන්නට තිබුණි. ගණිතය අතහැර දමා තිබූ සිසුන් ගණිත ගැටළු තේරුම් ගෙන විසඳීමට උත්සහ කරන බවක් නිරීක්ෂණය විය.

පාසල් පන්ති කාමරය තුළ ගණිතය ඉගැන්වීමට අධිප්‍රජානන උපාය මාර්ග භාවිතා කිරීමෙන්, අවබෝධයෙන් ගණිත ගැටළු විසඳීමේ හැකියාව වර්ධනය කළ හැකි බැවින්, පාසල තුළ ගණිතය ඉගැන්වීමට අධිප්‍රජානන උපාය මාර්ග භාවිතා කිරීමට යෝජනා කරනවා.

ගුරුවරයෙකු ලෙස මම පන්ති කාමරයේ සියලුම සිසුන්ට පුද්ගල භේද නොසලකා සමස්ත කණ්ඩායමක් ලෙස ඉගැන්වීමේ කටයුතු සිදු කළා.

එසේ කිරීම සෑම විටම සාර්ථක නොවන බවත්, දරුවන්ගේ මට්ටම හඳුනාගෙන ඔවුන්ගේ චින්තනය අවදි කරන ක්‍රියාකාරකම් සැලසුම් කිරීම වඩාත් සුදුසු බවත් මෙම කාර්යමූල පර්යේෂණය සිදු කිරීමෙන් මට වැටහුණා.

ඉදිරි පන්තිකාමර ඉගැන්වීමේදී මෙවැනි ක්‍රම භාවිතා කිරීමට බලාපොරොත්තු වෙනවා.

ඒ වගේම සිසුන්ගේ අවධානය උපරිම වශයෙන් දිනා ගත හැකිවන පරිදි හා සිසුන්ගේ චින්තනය අවධි වන පරිදි, ක්‍රියාකාරකම් සකස් කොට සිසුන්ට ප්‍රිය ජනක ඉගැනීමේ පරිසරයක් නිර්මාණය කිරීමට මට හැකි උපරිමයෙන් කටයුතු කරනවා.

ගණිත ගැටළු විසඳීම සඳහා අධිප්‍රජානනය භාවිතා කළ හැකි ආකාරය සහ සිසුන් සමඟ ප්‍රායෝගිකව පන්ති කාමරයක් තුළ අධිප්‍රජානන උපාය මාර්ග භාවිතා කළ හැකි ආකාරය මගේ වීඩියෝ එකෙන් බලාගන්න පුළුවන්.

ස්තූතියි...

Key Message 2- **Improving mathematical concept attainment through peer assisted learning.**

by Sandamali Mahakumbura

<u>සම වයස්ක සහයෝගී ඉගෙනීම තුළින් ගණිත සංකල්ප සාධනය ඉහළ නැංවීම</u>

ගුරුවරියක ලෙස ගණිතය ඉගැන්වීමේදී මට ඇති වූ ප්‍රධානම ගැටලුව වුණේ සාමාන්‍යපෙළ සිසුන්ගේ ගණිත සාධන මට්ටම ඉහළ නැංවීමට විශාල ප්‍රයත්නයන් දැරීමට සිදුවීම.

බොහෝ සිසුන් කණිෂ්ඨ ද්විතයික මට්ටමේ සිටම අඩු සාධන මට්ටම් පෙන්නුම් කරන බව ලකුණු නිරීක්ෂණ මගින් මට අවබෝධ වුණා.

මේ අතර මට අවස්තාවක් ලැබුණා පේරාදෙණිය විශ්ව විද්‍යාලයේ අධ්‍යාපන දෙපාර්තමේන්තුව මගින් ක්‍රියාත්මක කරනු ලබන සහයෝගී ක්‍රියා මූලික පර්යේෂණයට සහභාගී වීමට.

මෙහිදී 21 වන සියවසේ ඉගෙනුම් කුසලතා සංවර්ධනය පිළිබදවත්, පෙළපොත්වල සඳහන් ක්‍රියාකාරකම් සිසුන්ගේ ඉහළ ගණයේ චින්තන හැකියාව සංවර්ධනයට යොදාගත හැකි ආකාරය පිළිබදවත් අවබෝධයක් ලබා ගත්තා.

ඒ අනුව කණිෂ්ඨ ද්විතියික අංශයේ සිසුන්ට ක්‍රියාමූලික පර්යේෂණයක් කිරීමට අදහස් කළා.මේ සඳහා 7 ශ්‍රේණියේ සිසුන් තෝරා ගනු ලැබුවා.

පෙර පරීක්ෂණ ලකුණ / වාර විභාග ලකුණු/ඒකක පරීක්ෂණ ලකුණු නිරීක්ෂණය මගින් සිසුන්ගේ ගණිත සාධන මට්ටම පිළිබදව අවබෝධයක් ලබා ගත්තා.

සිසුන්ගේ ලකුණු මට්ටම අනුව 0-20/21 -40/41-60/61-80/81-100 අනුව සිසුන් කාණ්ඩ කළා

ඉගෙනුම් ඉගැන්වීම් වල යෙදෙන අතර එම සිසුන්ගේ චර්යාවන් නිරීක්ෂණය කිරීමටත්, සිසුන් සමග සුහදව කටයුතු කරමින් ඔවුන්ගේ ගණිත සාධනය පිළිබද ගැටලු තේරුම් ගැනීමට හැකි වුණා.

ගණිතය ඉතා අපහසු විෂයයක් ලෙස සැලකීම,
පෙළපොතේ අභ්‍යාස වල නොයෙදීම,
විෂය කරුණු නිතර නිතර අමතක වීම,
ගණිත සංකල්ප සාධනය කෙරෙහි බලපාන සාධක ලෙස හදුනා ගත්තා.

පළමු මැදහත්වීම ලෙස ගණිත විෂයය කරුණු ඉගෙනීමට පෙළඹවීමක් ඇති කිරීම

- ගණිත ක්‍රීඩා
- කණ්ඩායම් ක්‍රියා
- කේවල ක්‍රියාකාරකම්
- නිර්මාණාත්මක ක්‍රියා සැළසුම් කළා

ක්‍රියාකාරකම් අනුව ශිෂ්‍යය පෙළබවීමක් ඇති වුවත් පෙළ පොතෙහි අභ්‍යාස වල නිරතවීම අවම මට්ටමක පැවතුණා.

දෙවන මැදිහත්වීම ලෙස පෙළ පොතෙහි අභ්‍යාස වෙනුවට තේරීම, ඇදීම, හිස්තැන් පිරවීම වැනි සරල ක්‍රියාකාරකම් සැළසුම් කළා.

මෙම ක්‍රියාකාරකම් මගින් ගණිතය අපහසු විෂයයක් නොවන බව සිසුන්ට අවබෝධ කරවීමට මම උත්සාහ කළා. ඉන්පසුව පෙළපොතෙහි අභ්‍යාස සඳහා සිසුන් යොමු කළා.

පසු පරීක්ෂණ මගින් සිසුන්ගේ සාධන මට්ටම් සොයා බැලූ අතර, ඒ අනුව ඉදිරි මැදහත්වීම් සැළසුම් කළා.

පරීක්ෂණ අවසානයේ සිසුන් සිටින කාණ්ඩය පිළිබඳව ඔවන්ට අවබෝධයක් ලබා ගැනීමට ඉඩ සැලසූ අතර කළ අතර අවම වශයෙන් තමා සිටින මට්ටමට වඩා ලකුණු 5ක් වත් ලබා ගැනීමට සිසුන් දිරිමත් කළා.

පහල සාධන මට්ටම්වල සිටි සිසුන්ට ගැටලු කියවා තේරුම් ගැනීමට අපහසු බවත් මූලික ගණිත කර්ම පිළබඳ ගැටලු ඇති බවත් අවබෝධ වුණා.

මීලග මැදිහත්වීම ලෙස 21 වන සියවසේ ඉගෙනුම් කුසලතා සංවර්ධනය සඳහා යොදා ගත හැකි සම*වයස්ක* සහයෝගී ඉගෙනුම (peer assisted learning) පිලිබඳ සංකල්පය යොදා ගත්තා.
මේ යටතේ

- සිසුන් කණ්ඩායම් කර සහයොගයෙන් විසඳීමට ගැටලු ලබා දීම
- ගණිත දැලිස වැඩසටහන
- ඉහල සාධන මට්ටම් වල සිසුන්ට පහළ සාධන මට්ටම්වල සිසුන් තෝරා දීම සිදු කළා

මෙහිදී සිසුන්

- පුණරීක්ෂණ අභ්‍යාස ලබාදීම
- පෙළපොතෙහි ගැටලු විසඳීමට සහයෝගය ලබාදීම
- *Zoom class* පැවැත්වීම වැනි

විවිධ උපක්‍රම භාවිතකරමින් පහළ සාධන මට්ටම්වල සිසුන්ට සහයෝගය දැක්වීමට පෙළඹුණා.

ක්‍රමයෙන්, ඒකක පරීක්ෂණ සඳහා සිසුන් පෙරට වඩා වැඩි උනන්දුවක් දක්වන බව නිරීක්ෂණය කළ අතර, සාධන මට්ටම වැඩි කර ගැනීමට සිසුන්උත්සාහ කරන බව දක්නට ලැබුණා.

ක්‍රියාකාරකම් මගින් ගණිත සංකල්ප සාධනය කිරීමට ගත් උත්සාහය නිසා, ගණිතය විෂයය ඉගෙනීමට සිසුන් තුළ අභ්‍යන්තර පෙළඹවීමක් ඇතිවුණා.

සමවයස් සිසුන් සමග සහයෝගීව ඉගෙනීමට යොමු කිරීම තුළින්, පහළ සාධන මට්ටම්වල සිසුන්ගේ හැකියාවන් මෙන්ම ඔවුන්ට සහය වූ ඉහළ සාධන මට්ටම්වල සිසුන්ගේ හැකියාවන්ද සංවර්ධනය වූ බව දක්නට ලැබුණා.

මාගේ පර්යේෂණයට අනුව පන්ති කාමරය තුළ ක්‍රියාකාරකම් හා සහයෝගී ඉගෙනුම් අවස්ථා ලබාදීමට සැළසුම් කළ පාඩමක් මාගේ වීඩියෝවෙන් ඉදිරිපත් කරනවා.

ස්තුතියි...

Key Message 3- **Use of collaborative problem solving in mathematics classrooms.**

by Darshani Herath

<u>ගණිත පන්ති කාමරය තුළ සහයෝගී ගැටළු විසඳීම භාවිතා කිරීම.</u>

මම දර්ශනී හේරත් රජයේ පාසලක ගණිත ගුරුවරයකු ලෙස අවුරුදු නවයක කාලයක් තිස්සේ මම සේවය කරනවා.

ගණිතය ගැන්වීමේදී මට ඇතිවුණ ප්‍රධාන ගැටලුවක් තමයි, ගුරුවරුන් ලෙස කොපමණ වෙහෙසක් ගත්තත් සිසුන්ගෙන් අපේක්ෂිත සාධන මට්ටම් හරියාකාරව ලැබෙන්නේ නැති කම.

ඒවගේම සිසුන් ගුරුවරයා සමඟ සක්‍රීය ලෙස පාඩමට සම්බන්ධ නොවීම.

මේ අතර මට අවස්ථාව ලැබුණා, පේරාදෙණිය විශ්වවිද්‍යාලයේ අධ්‍යාපන දෙපාර්තමේන්තුව මගින් ක්‍රියාත්මක කරනු ලබන මධ්‍යම පළාතේ කනිෂ්ඨ ද්වීතික අංශයේ ගණිත සාධනය පිළිබඳව සහයෝගී ක්‍රියාමූලික පර්යේෂණයකට සහභාගී වෙන්න.

එහිදී කනිෂ්ඨ දිවිතීක අංශයේ පවතින ගැටලුවක් හඳුනාගැනීම සඳහා සිසුන්ට ප්‍රශ්නවලියක් ලබා දුන්නා.

එසේම එම ප්‍රශ්ණාවලිය සඳහා සිසුන්ගේ අදහස් අනුව සලකා බලමින් හඳුනාගත් ගැටලුවක් නිරාකරණය කිරීම සඳහා ක්‍රියා මූලික පර්යේෂණයක් සිදු කිරීමට අවස්ථාව ලැබුණා.

එම ප්‍රශ්ණාවලිය සඳහා සිසුන් ලබාදුන් පිළිතුරු සලකා බැලීමෙන් මට හැඟී ගියේ සිසුන් 80% ක් පමණ ගණිතය ඉතාමත් ම අමාරු විෂයයක් ලෙස සලකන බව.

ඔවුන්ගේ ප්‍රියතම විෂයය ගණිතය නොවන බව.

තවද එම පර්යේෂණයට සමගාමීව විසි එක්වන සියවසේ කුසලතාව සහ එහි භාවිතයන් පිළිබඳව ගුරුවරුන් දැනුවත් කළා.

එම දැනුම උපයෝගී කරගෙන මට ප්‍රධාන ගැටලුවක්ව තිබෙන ළමුන්ගේ සාධන මට්ටම් ඉහළ දැමීම සඳහා යමක් කරන්න පුළුවන් වේවි කියලා මට හිතුනා.

පසුව පර්යේෂණයට අදාළ පුහුණු සැසි වාර වල දී ඉගෙනගත් 21 වන සියවසේ කුසලතා වලින් ප්‍රධාන වශයෙන් කතා කරන සහයෝගී ඉගැන්වීමට අදාළව ක්‍රියාමූලික පර්යේෂණය සැලසුම් කරා.

සහයෝගී ඉගැන්වීම් තුලදී; සිසුන් දෙදෙනෙකු හෝ වැඩි ගණනක් ඔවුන්ගේ දැනුම, කුසලතා සහ උත්සාහයන් එකතු කරමින් සාර්ථක විසඳුම් ලබා ගන්නා අවස්තා පිලිබඳව සහ එහිදී තනි සිසුවෙක් දියුනු කරගන්නා කුසලතා පිලිබඳව කතා කරනවා.

එදිනෙදා ජීවිතයේ හමුවන *ප්‍රායෝගික* ගැටලු ආශ්‍රයෙන් සැකසූ ගණිත ගැටළු, සහයෝගීව විසඳීම සඳහා මම සිසුන්ට අවස්ථාව ලබා දුන්නා.

මාස තුනක පමණ කාලයක් තිස්සේ මම ක්‍රියා මූලික පර්යේෂණය සිදුකළා.

ක්‍රියා මූල පර්යේෂණයට අදාළ තක්සේරු මගින් සිසුන්ගේ සාධන මට්ටම් වල වර්ධනයක් ඇති වූ බව පෙනුනා.

ගණිතය පිළිබඳ උනන්දුව වැඩි වුණ ආකාරයක්, සහ සිසුන් ඉතා සහයෝගයෙන් පිළිතුරු ලබාගන්නා ආකාරයත් මට පන්ති කාමරයේ දී දක්නට ලැබුණා.

ඒ වගේම සිසුන්ගේ *සමාජ කුසලතා* වර්ධනයත්;

සැබෑ ජීවිතයේදී ඇතිවන ගැටළු නිරාකරණය කිරීම සඳහා ගණිතය අවශ්‍ය බවත් සිසුන් අවබෝධ කරගන්නවා.

ක්‍රියාමූලික පර්යේෂණ සිදුකරමින් යන අතර සහයෝගීතාවය වැඩි කිරීමේ හැකියාව ඇති ක්‍රියාකාරකම් යොදා ගැනීම සඳහා අන්තර්ජාලය පරිහරණය කළා. ගුරු මාර්ගෝපදේශය සංග්‍රහයට බාහිරව යමින් අලුත් ක්‍රියාකාරකම් හඳුන්වා දුන්නා.

සාමාන්‍ය පන්ති කාමරයේ දී සිදු කරනවාට වඩා සිසුන්ට නිවසේදී තාක්ෂණය භාවිතයෙන් අලුත් දැනුම් සොයා ගැනීම්ට අවස්ථාව ලබාදීමෙන්,

මෙවැනි සහයෝගී ක්‍රියාකාරකම් සිදු කිරීම නිසා සිසුන් තුළ ගණිතය සඳහා පෙළඹවීම වැඩිවෙන බව මට පෙනී ගියා.

මෙමගින් ගුරුවරියක වශයෙන් මාගේ වෘත්තීය දැනුම හා කුසලතා ද වැඩිදියුණු කර ගැනීමට අවස්ථාව ලැබුනා.

ඉදිරියේදිත් පන්ති කාමරය තුළ දී මෙවැනි සහයෝගී ගැටලු විසඳීමේ අවස්ථා දිගින් දිගටම සිසුන්ට ලබාදීමට මම අදහස් කරනවා.

එම ක්‍රියා මූලික පර්යේෂණය සඳහා සිසුන්ට ලබාදුන් වර්ගඵලය පාඩම ආශ්‍රිතව ගණිත ගැටලුවක් සහයෝගී ආකාරයෙන් විසඳීම සඳහා සිසුන් යොමුවූ අයුරු මගේ වීඩියෝවෙන් බලාගන්න පුලුවන්
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ස්තුතියි...

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Key Message 4- **Use of collaborative problem solving in mathematics classrooms.**

by Sulakshi Gunasinghe

<u>රූප සටහනක් ඇසුරින් , ඉහළ ප්‍රජානන මට්ටමේ ගණිත ගැටලු විසඳීම</u>

පාසල් ගුරුවරියක ලෙස, ගණිතය විෂය ඉගැන්වීමේදී නිරීක්ෂණය වූ කරුණක් වූයේ සිසුන් සරල මට්ටමේ ගැටලු උනන්දුවෙන් හා ඉක්මනින් නිම කරත් තරමක් සංකීර්ණ ගැටලු තේරුම් ගැනීමේ අපහසුතා නිසා නිතර ප්‍රශ්න කරමින් වැඩි කාලයක් ගත කරන බවක් .

ඒ කියන්නේ ඉහළ ප්‍රජානන මට්ටමේ ගැටලු විසඳීමට සිසුන් තුළ අපහසුතා පවතිනවා.

මෙම ගැටලුව හඳුනාගැනීම සිදු වූයේ , ගණිතය විෂය ඉගෙනීමේදී ,සිසුන් පංතිකාමරයේදි ගනිත ගැටලු විසඳීම උනන්දුවෙන් සිදුකලත් පාඩම අවසානයේදි දෙන ගෙදර වැඩ (පසුවැඩ) නිතර අතපසු

කරන බවක් නිරීක්ෂණය විම නිසා. එම නිසා ගෙදර වැඩ අවම කරලා , පංති කාමරයේදි වැඩි ගැටලු ප්‍රමාණයක් විසදීමේ අවස්තාව සිසුන්ට ලබාදීම කෙරෙහි මගේ අවධානය යොමු වෙනවා.

පංතියේදි වැඩි ගැටලු ප්‍රමාණයක් විසදීමට නම්, සිසුන්ගේ ගණිතය ගැටලු තේරුම් ගැනීම හා විසදීමේ වේගය වැඩි විය යුතු උනා.ඒ කියන්නේ සිසුන්ගේ ගණිත කායීක්ෂමතාවය ඉහළ මට්ටමට ගෙන ආ යුතු උනා .

පාසල් ගුරුවරියක ලෙස, ගණිතය විෂය ඉගැන්වීමේදී නිරීක්ෂණය වූ කරුණක් වූයේ සිසුන් සරල මට්ටමේ ගැටලු උනන්දුවෙන් හා ඉක්මනින් නිම කරත් තරමක් සංකීර්ණ ගැටලු තේරුම් ගැනීමේ අපහසුතා නිසා නිතර ප්‍රශ්න කරමින් වැඩි කාලයක් ගත කරන බවක් .

ඒ කියන්නේ ඉහළ ප්‍රජානන මට්ටමේ ගැටලු විසදීමට සිසුන් තුළ අපහසුතා පවතිනවා.

මෙම ගැටලුව හඳුනාගැනීම සිදු වූයේ , ගණිතය විෂය ඉගෙනීමේදී ,සිසුන් පංතිකාමරයේදි ගනිත ගැටලු විසදීම උනන්දුවෙන් සිදුකලත් පාඩම අවසානයේදි දෙන ගෙදර වැඩ (පසුවැඩ) නිතර අතපසු කරන බවක් නිරීක්ෂණය විම නිසා. එම නිසා ගෙදර වැඩ අවම කරලා , පංති කාමරයේදි වැඩි ගැටලු ප්‍රමාණයක් විසදීමේ අවස්තාව සිසුන්ට ලබාදීම කෙරෙහි මගේ අවධානය යොමු වෙනවා.

පංතියේදි වැඩි ගැටලු ප්‍රමාණයක් විසදීමට නම්, සිසුන්ගේ ගණිතය ගැටලු තේරුම් ගැනීම හා විසදීමේ වේගය වැඩි විය යුතු උනා.ඒ කියන්නේ සිසුන්ගේ ගණිත කායීක්ෂමතාවය ඉහළ මට්ටමට ගෙන ආ යුතු උනා .

මේ අතරතුර පේරාදෙණිය විශ්ව විද්‍යාලයේ අධ්‍යාපන අංශය මගින් සිදුකරන පර්යේෂණයකට සහභාගී වෙන්න අවස්තාව මට ලැබෙනවා

මෙම පර්යේෂණ කණ්ඩායම සමග , ඉගැන්වීම් ක්‍රියාවලිය තුළ පවතින ගැටලුවක් නිරාකරණය කරගැනීම සදහා ක්‍රියාමූලික පර්යේෂණයක් සිදුකිරීම *සදහා මට අවස්තාව ලැබුණා*

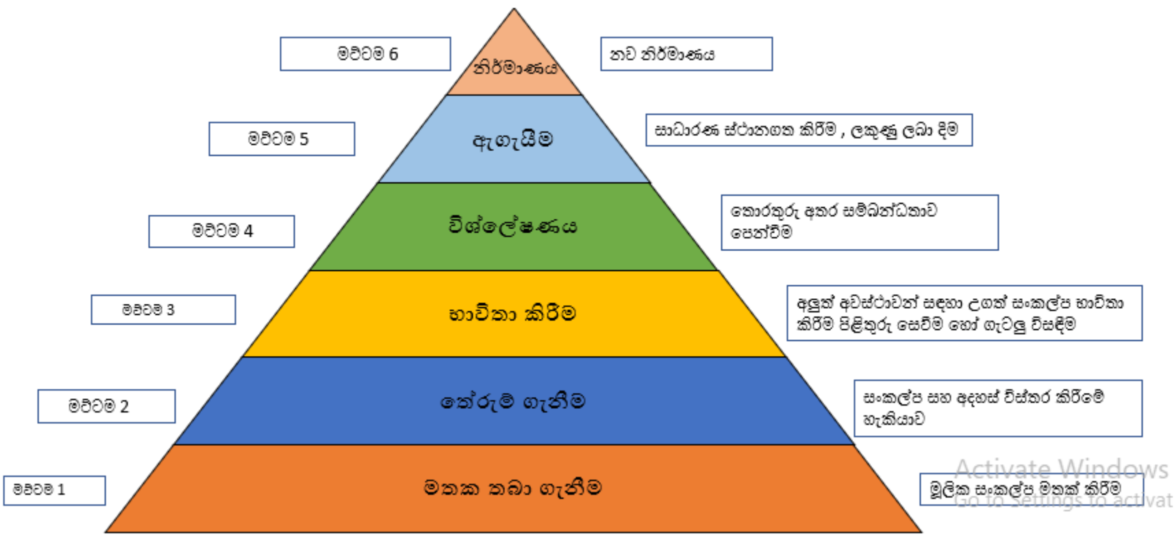
මෙම පර්යේෂණ දැනුවත් කිරීම්වලදි ලබාදුන් දේශන අතරින් විසිඑක්වන සියවසේ කුසලතා හා එහි භාවිතාවන් පිලිබද දැනුවත් කිරීම් , විවේචනාත්මක චින්තනය ,*බ්ලූම්ගේ වර්ගීකරණ දැනුම* මට වඩා වැදගත් උනා.

ඒ අනුව සිසුන් තුල පවතින ඉහළ ප්‍රජානන මට්ටමේ ගැටලු විසදීමේ අපහසුතා නිරාකරණය කරගැනීමට , පර්යේෂණය කණ්ඩායමෙන් ලද දැනුම ද යොදාගෙන *මැදිහතවීම සිදුකළා.*

බ්ලූම්ගේ *වර්ගීකරණය අනුව මූලික දැනුම ලබාදීමෙන් පසු* ,මතක තබාගැනීමට හා අවබෝධයෙන් භාවිතා කිරීමට *අවස්තා ලබා දීම අත්‍යවශ්‍ය වේ.සිසු කේන්ද්‍රය අභ්‍යාස මේ අවස්තා ශිෂ්‍යයන්ට ලබාදෙයි.*

(රූපය 01)

බ්ලූම්ගේ වර්ගීකරණය



සරල මට්ටමේ සිට සංකීර්ණ කරුණු දක්වා පරාසය අනුගමනය කරමින් නිවැරදි වනතුරු ඉගැන්වීම මගින් ශිෂ්‍යයන් සාර්ථකත්වය කරා ගෙන යා හැකි බව ද බ්ලූම් පැහැදිලි කරයි.

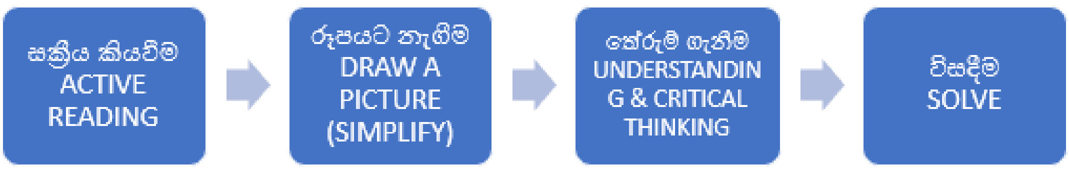
මේ අනුව පංතිකාමරයේදි එදිනෙදා පාඩම්වලට සමගාමීව අභ්‍යාස ලබා දීමේදී , සරල ගැටලු විසඳීමෙන් මැදිහත්වීම ආරම්භ කෙරුණා.

මේ මැදිහතවීම අතරින් රූප සටහනක් භාවිතයෙන් ගනිත ප්‍රශ්නයක් සාකච්ඡා කිරීමේදි වඩා පහසුවෙන් ප්‍රශ්නය අවබෝධ කරගන්නා බවක් සිසුන්ගෙන් නිරීක්ෂණය වුනා.

ඒ අනුව රූප සටහන් යොදාගෙන ගැටලු විසදීමේ අවස්තා වැඩිවශයෙන් පංති කාමරයේදි අත් හදා බැලුනා.

සමහර අවස්තාවල ගැටලුව නිරවුල්ව තේරුම් ගනීමට කටු සටහනක් පවා ප්‍රමාණවත් වුනා.

ගනිත ප්‍රශ්නයක් සාකච්ඡා කිරීමේදි ගැටලුවට අදාළ රූපසටහනක් ගොඩනැගීමට ,පියවර 4 ක ක්‍රමවෙදයක් අනුගමනය කෙරුණා. (රූපය02)



කාර්ය මූල පර්යේෂණයට අදාළ පසු ඇගැයීම් මගින් ,රූප සටහන් භාවිතයෙන් ගැටලු විසදීමේ දී අවබෝධයෙන් ගැටලුව විසදීමේ යොමුවීමක් සිසුන් තුළ ඇති වී තිබෙන බව නිරීක්ෂණය වුනා.

,නිවැරැදි අවසන් පිළිතුර ලබා ගැනීමට සෑම විටම සිසුන්ට නොහැකි වුවත්, පෙර අවස්ථාවලදීට සාපේක්ෂව , ගැටළුව කියවීම පවා මගහැර සිටි සිසුන්, ප්‍රජානන මට්ටම් වර්ගීකරණය අනුව 1 , 2 සහ ඊට ඉහළ ප්‍රජානන මට්ටම් වල ගැටළු විසඳීමේදී,අවම වශයෙන් ගැටලුවේ අර්ධයක් හෝ විසඳෙන මට්ටමට දියුණුවක් නිරීක්ෂණය වුණා.

මෙම පර්යේෂණය සිදු කිරීමෙන් මා ලද අත්දැකීම් අනුව මට හැගී ගියෙ , සිසුන් තුල , ගැටලු තේරුම් ගැනීමෙ හැකියාව දියුණු කල හැකි නම්, ඉහළ ප්‍රජානන මට්ටමේ ගැටලු විසදීමට සිසුන් යොමු වන බවයි.

ඒසේම යම් ගැටලුවක අඩංගු විශේෂ කරුණු ,සරල රූපයකට නැගීමෙන් එම ගැටලුව තේරුම් ගැනීම පහසු කරන බවකි. ඉන්

පංති කාමරය තුලදී සංකීර්ණ ගැටලු විසදීමේ නැඹුරුව සමග පාසල් කාලයෙන් පසු ,නිවසේදී පෙළ පොතේ අභ්‍යාස සම්පූර්ණ කිරිමේ ස්වයං නැඹුරුවක් සිසුන් තුල ඇති වුණා.

මෙම ගෙදර වැඩ සම්පූර්ණ කිරීමේ චර්යාත්මක වර්ධනය පවත්වා ගැනීමට හා උත්ප්‍රේරනය සඳහා ධනාත්මක උපස්තම්භනය යොදා ගත්තා.එය වඩාත් හොද උනන්දුවක් සිසුන් තුල ඇති කලා.

ඉන්පසු , පාසල් කාලය තුල විවේචනාත්මක චින්තනය වර්ධනට යොමුකරවන ගණිතමය ක්‍රියාකාරකම් භාවිතා කිරීම වැනි උපක්‍රම යොදා ගත්තා.ඉන් ගණිතය සඳහා පෙලඹවීම වැඩි වුනා.

සිසුන්ගේ වියුක්ත චින්තනය අවදි කිරීම සිදුවෙන අතර , එමනිසා රූපසටහනක් යොදා ගනිමින් ගැටලුව විසදීමට උත්සාහ කිරීමෙන් පහසුවෙන් විසඳුම හදුනාගැනීමට අවස්තාව ලැබේ.

මෙ අනුව පංති කාමර ඉගැන්වීම් වලදි අවශ්‍ය අවස්ථාවල රූපසටහනක් භාවිතයෙන් ගනිත ගැටලු විසදීම උචිත බව මම අදහස් කරනවා.

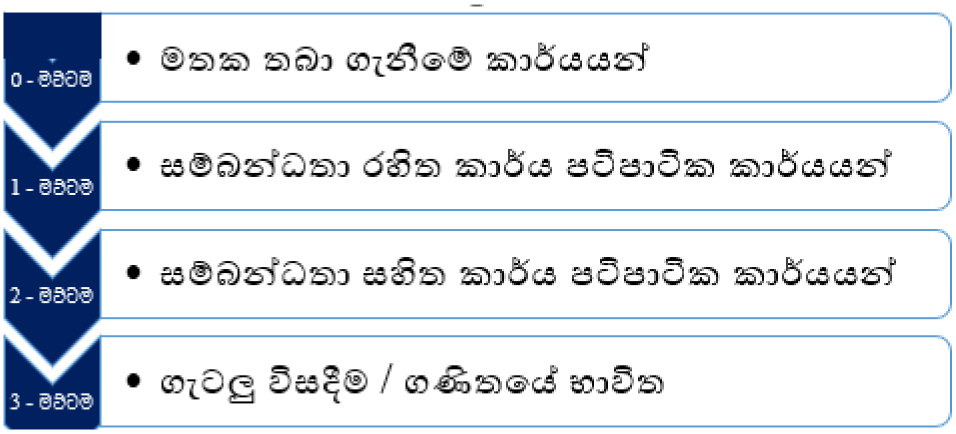
පංති කාමරයේදි යොදාගත් , රූප සටහන් ආශ්‍රයෙන් ගනිත ප්‍රශ්න විසදීමට සිසුන් යොමු කරවීමේ අවස්තාවක් මගේ වීඩියෝවෙන් බලමු.

Key Message 5- Improving students' mathematical problem solving skills through creative strategies.

by Prabha Lochani Adhikaram

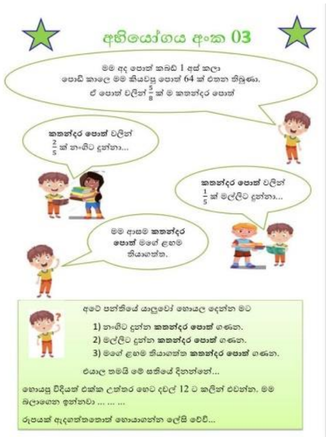
<u>සිසුන්ගේ, ගණිත ගැටලු විසදීමේ හැකියාව නිර් මාණශීලී උපක්‍රම ඇසුරින් දියුනු කිරීම</u>

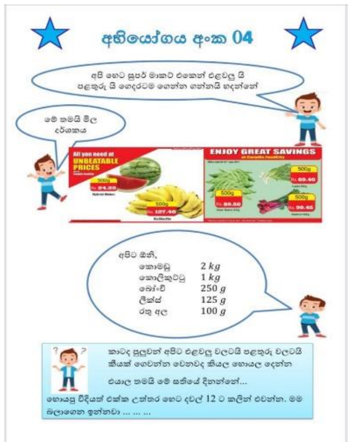
- මම ලෝචනී අදිකාරම්, රජයේ පාසලක ගණිත ගුරුවරියක් විදියට අවුරුදු 6 ක කාලයක් තිස්සේ මම සේවය කරනවා.
- ගණිතය විෂයේ ඉ/ඉ ක්‍රියාවලිය තුල යෙදෙන විට මට ඇතිවුන ගැටලුවක් තමයි පාඩම අවසානයේ ලබා දෙන අභ්‍යාස වලදී ක්ෂණිකව පිළිතුරු සපයන්න පුලුවන් ප්‍රශ්න වලට පිළිතුරු සැපයුවත් අවබෝධයෙන් විසඳිය යුතු ගැටලු වලට පිළිතුරු සපයන්න සිසුන් යොමුනොවීම.
- ඒ වගේම තමයි යාන්ත්‍රිකව පිළිතුරු සපයන්න ද සිසුන් යොමු වෙලා ඉන්නව.
- මේ අතරදී පේරාදෙණිය විශ්ව විද්‍යාලයේ අධ්‍යාපන විද්‍යා අධ්‍යයන අංශය මගින් ක්‍රියාත්මක කරපු AHEAD ව්‍යාපෘතිය යටතේ පැවැත්වුන වැඩමුලු වලට සහභාගී වෙන්න මට අවස්ථාවක් ලැබුනා.
- ඒ වැඩමුලු වලදී කාර්යමූලික පර්යේෂණ ගැන පුලුල් දැනුමක් ලැබුණා සහ අවසානයේ විශ්වවිද්‍යාල ආචාර්ය වරුන්ගේ අධීක්ෂණය යටතේ කාර්යමූලික පර්යේෂණයක් කරන්න අවස්ථාවක් ලැබුනා.
- මගේ කාර්යමූලික පර්යේෂණය තුලින් අවබෝධයෙන් විසඳිය යුතු ගණිත ගැටලු වලට පිළිතුරු සපයන්න සිසුන් යොමුනොවීම යන ගැටලුවට උත්තර හොයන්න මම හිතුවා.
- මම වැඩමුලු වලින් ඉගෙනගත්ත ගණිතමය කාර්යයන්හි ප්‍රජානන ඉල්ලුම් මට්ටම් කියන සංකල්පය සහ 21 වන සියවසේ කුසලතා ගැන දැනුම මගේ කාර්යමූලික පර්යේෂණය කරන්න ගොඩක් උදව් උනා.
- ගණිතමය කාර්යයන්හි ප්‍රජානන ඉල්ලුම් මට්ටම් කියන සංකල්පය ගැන කෙටියෙන් කියනවනම්, මේකෙ මට්ටම් 4 ක් තියෙනව, ඒව 0,1,2,3 විදියට කොටස් කරල තියෙනව,



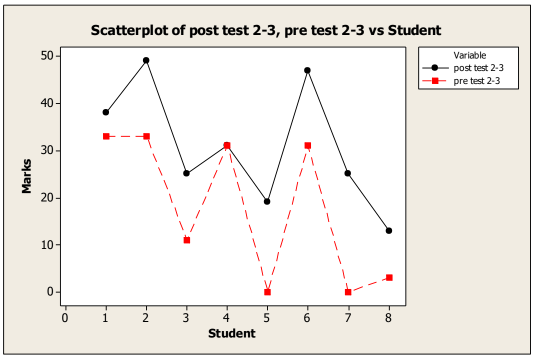
- ඒ අනුව මෙම කාර්යමූලික පර්යේෂණයේ දී ගණිතමය කාර්යයන්හි ප්‍රජානන ඉල්ලුම් මට්ටම් 2 සහ 3 යටතේ ඇති ගණිත ගැටලු විසඳීම ට අදාලව සිසුන්ගේ සාධන මට්ටම ඉහල නැංවීම මගේ අරමුණ උනා.
- ඒ නිසා ක්‍රියාකාරකම් දෙනකොට විවේචනාත්මක චින්තනය, ගැටලු විසඳීම, සංනිවේදනය, සහයෝගීතාව හා අධිප්‍රජානනය යන අංශ ඔක්කොම ඇතුලත් වෙන විදියට ක්‍රියාකාරකම් සකස් කලා.

- සාමාන්‍ය ගණිත ගැටලුවම නිර් මාණශීලීව ගොඩනගල සහ උපක්‍රමශීලීව ලබාදීල අවබෝධයෙන් විසදිය යුතු ගැටලු වලට පිළිතුරු සපයන්න සිසුන් යොමු කරගන්න මම උත්සාහ කලා.





- මගේ කාර්යමූලික පර්යේෂණය මාස තුනක් පමණ සිදුකලා.
- කාර්යමූලික පර්යේෂණය අවසානයේ අවබෝධයෙන් විසදිය යුතු ගණිත ගැටලු වලට පිළිතුරු සපයන්න සිසුන්ගේ යොමු වීම සැලකිය යුතු ලෙසත් සාධන මට්ටම යම් තරමකිනුත් ඉහල ගිහින් ඇති බව අදාල තක්සේරු වලින් පෙනී ගියා.
- ඒ වගේම තමයි විෂයට පරිබාහිරව විෂය සමගාමී ක්‍රියාකාරකම් වලටද පන්තියේ සිසුන්ගේ සහභාගී වීම අනෙක් පන්ති වලට සාපේක්ෂව ඉහල ගොස් ඇති බව දැනටත් නිරීක්ෂණය වෙනව.



- ඇත්තටම මෙම කාර්යමූලික පර්යේෂණය නිසා ගුරුවරියක ලෙස මගේ වෘත්තීය දැනුම හා කුසලතා ඉහල නංවා ගැනීමට මට අවස්ථාව ලැබුනා.
- මම කාර්යමූලික පර්යේෂණය සමස්ථ පන්තියටම අදාල කාලච්ඡේද තුල පමණක් සිදු කල නිසා මෙම කාර්යමූලික පර්යේෂණයේදී යොදාගත් ආකාරයේ ක්‍රියාකාරකම් ඉදිරියටත් භාවිතා කරන්න මම අදහස් කරගෙන ඉන්නව.
- කාර්යමූලික පර්යේෂණයේ දී, අවබෝධයෙන් විසදිය යුතු ගණිත ගැටලු විසදීමට සිසුන් යොමු කර ගන්නා ආකාරය මගේ මීලග වීඩියෝ එකෙන් බලාගන්න පුලුවනි.
- පන්ති කාමරයේ ඉ/ඉ ක්‍රියාවලිය තුලදීම අවබෝධයෙන් විසදිය යුතු ගණිත ගැටලු විසදීමට සිසුන් යොමු කර ගන්නා ආකාරය ගැන ඔබ සැමට අවබෝධයක් ලැබෙන්න ඇතැයි මම විශ්වාස කරනවා.

ඉතින් මේ ක්‍රමවේදය පන්ති කාමරය තුල දී අත්හදා බලන්න කියල මම, ඔබ සැමට ආරාධනා කරනවා.

Key Message -6 Thishani Bandaranayake

Hello everyone. I am Thishani Bandaranayake who is working as a mathematics teacher in government sector for 7 years. In my career life I have seen students face difficulties when answering essay type questions compared to MCQ type questions. And at first glance they decide its' difficult to solve essay type problems. With further observations and interviews it could figure out students are lack of critical thinking and self-confidence towards mathematics. While I was trying to find a solution for my problem, I got a chance to work as a research assistant at world bank funded AHEAD project which was coordinated by department of Education, at University of Peradeniya. We had seminars and lectures on educational research. So, there I could understand action research is the best way to solve my problem. And with early research I could finalize that use of blended learning along with inquiry-based learning will be beneficial for my study. Mathematics is a vast area and geometry is a sub field of mathematics and in it Cartesian geometry become an important component. In Sri Lankan mathematics education system Cartesian plane starts with grade 7, so it was selected as the sample for my lesson. In the lesson group activity was given for each group where they must do a task using cartesian plane. It could identify students engagement high, asked questions frequently and with the questioned they asked they got the knowledge which they needed to do the task in cartesian plane. With further interviews and observations, it could figure students self-confidence was built up as they did it by themselves. As well as I could observe that some components of critical thinking were developed as inference, interpret, self-organization and etc, hence critical thinking is a vast skill, it can't be developed with in one day. As per my experience I could clearly state that it is beneficial to use these kinds of activities in the mathematics lesson rather than traditional classroom. So going to improve this method in my future lessons and I invite all mathematics teachers to use these kind of lesson approaches and methods in there mathematics lessons.